Elementary Education: Mathematics—CKT
Welcome to The Praxis® Study Companion

Prepare to Show What You Know

You have been working to acquire the knowledge and skills you need for your teaching career. Now you are ready to demonstrate your abilities by taking a Praxis® test.

Using the Praxis Study Companion is a smart way to prepare for the test so you can do your best on test day. This guide can help keep you on track and make the most efficient use of your study time.

The Study Companion contains practical information and helpful tools, including:

• An overview of the Praxis tests
• Specific information on the Praxis test you are taking
• A template study plan
• Study topics
• Practice questions and explanations of correct answers
• Test-taking tips and strategies
• Frequently asked questions
• Links to more detailed information

So where should you start? Begin by reviewing this guide in its entirety and note those sections that you need to revisit. Then you can create your own personalized study plan and schedule based on your individual needs and how much time you have before test day.

Keep in mind that study habits are individual. There are many different ways to successfully prepare for your test. Some people study better on their own, while others prefer a group dynamic. You may have more energy early in the day, but another test taker may concentrate better in the evening. So use this guide to develop the approach that works best for you.

Your teaching career begins with preparation. Good luck!

Know What to Expect

Which tests should I take?

Each state or agency that uses the Praxis tests sets its own requirements for which test or tests you must take for the teaching area you wish to pursue.

Before you register for a test, confirm your state or agency’s testing requirements at www.ets.org/praxis/states.

How are the Praxis tests given?

Praxis tests are given on computer. Other formats are available for test takers approved for accommodations (see page 43).
What should I expect when taking the test on computer?

When taking the test on computer, you can expect to be asked to provide proper identification at the test center. Once admitted, you will be given the opportunity to learn how the computer interface works (how to answer questions, how to skip questions, how to go back to questions you skipped, etc.) before the testing time begins. Watch the What to Expect on Test Day video to see what the experience is like.

Where and when are the Praxis tests offered?

You can select the test center that is most convenient for you. The Praxis tests are administered through an international network of test centers, which includes Prometric® Testing Centers, some universities, and other locations throughout the world.

Testing schedules may differ, so see the Praxis web site for more detailed test registration information at www.ets.org/praxis/register.
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1. Learn About Your Test

Learn about the test you will be taking

Elementary Education: Mathematics—CKT (7803)

Test at a Glance

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<th>Elementary Education: Mathematics—CKT</th>
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<tr>
<td>Test Code</td>
<td>7803</td>
</tr>
<tr>
<td>Total Time</td>
<td>85 minutes</td>
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<tr>
<td>Format</td>
<td>Selected-response and numeric-entry questions; on-screen four-function calculator provided</td>
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<table>
<thead>
<tr>
<th>Content Categories</th>
<th>Approximate Number of Questions</th>
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<tr>
<td>I.  Counting and Operations with Whole Numbers</td>
<td>16</td>
<td>30%</td>
</tr>
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<td>13</td>
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<tr>
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<td>13</td>
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</tr>
<tr>
<td>IV. Early Equations and Expressions, Measurement and Geometry</td>
<td>10</td>
<td>20%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>52</strong></td>
<td><strong>100%</strong></td>
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About This Test

This test focuses on the essential content knowledge needed for teaching elementary mathematics. The 47 one-point questions and 5 two-point questions measure two kinds of content knowledge. The first kind is the content knowledge needed to do the work of the student curriculum, such as solving a math problem similar to one students would solve. Approximately 20 percent of the questions measure this kind of content knowledge. The second kind is the specialized content knowledge needed to teach the student curriculum; this is the knowledge a teacher would use, for instance, when choosing an example to demonstrate a mathematical concept or interpreting a student’s mathematical misunderstanding based on a pattern of errors. This specialized content knowledge is not knowledge that students are expected to learn, nor is it general knowledge of classroom-management strategies or learning theory; it is mathematical knowledge specialized to the work of teaching elementary mathematics. Approximately 80 percent of the questions measure this kind of content knowledge.

Each question focuses on specific mathematics content (listed in “Content Topics”). Questions measuring specialized content knowledge also incorporate a particular task of teaching mathematics (listed in “Tasks of Teaching Mathematics”); these questions are intended to measure the specialized content knowledge needed to carry out the practice effectively.

This test may contain some questions that will not count toward your score.

This test was developed through a partnership between the Educational Testing Service and TeachingWorks at the University of Michigan. It draws on the theoretical framework of content knowledge for teaching, grounded
in over 25 years of research, which identifies a type of professional content knowledge used only in teaching. Research evidence links this specialized content knowledge to improved content teaching and to positive learning outcomes for students. For more information about TeachingWorks, see [http://www.teachingworks.org/](http://www.teachingworks.org/). For a CKT overview, see Deborah Loewenberg Ball, Mark Hoover Thames, and Geoffrey Phelps, “Content Knowledge for Teaching: What Makes It Special?” Journal of Teacher Education 59, no. 5 (2008): 389-407.

Below are descriptions of the three sections that follow: “Tasks of Teaching Mathematics,” “Content Topics,” and “Measuring Content Knowledge in Practice.”

“Tasks of Teaching Mathematics” lists sixteen tasks that are a routine part of elementary mathematics instruction. These tasks are based in part on the mathematical work that teachers must do to be able to implement certain high-leverage practices (HLPs) identified by TeachingWorks. They were identified by ETS and TeachingWorks, with input from a national committee of elementary teachers and teacher educators, as being among the most essential tasks for effective teaching of elementary mathematics content.

“Content Topics” is a list of critical mathematics content that students are expected to master at the elementary level. The list was derived from student standards for elementary mathematics. The topics included were confirmed as important through a national survey of the field and refined by a national committee of elementary teachers and teacher educators.

“Measuring Content Knowledge in Practice” provides a more detailed explanation of the relationship between the content topics and tasks of teaching in the test. It also provides a sample test question to illustrate this relationship.

**On-Screen Calculator**

An on-screen calculator is provided for the computer-delivered test.

![On-Screen Calculator](image)

Please consult the web page [Praxis Calculator Use](http://www.teachingworks.org/) for further information and [review the directions for using the on-screen calculator](http://www.teachingworks.org/).

**Tasks of Teaching Mathematics**

This list includes tasks that are essential for effective teaching of elementary mathematics.

**Explanations, Conjectures, and Definitions**

1. Giving mathematically valid explanations for a process, conjecture, or relationship
2. Evaluating mathematical explanations for their validity, generalizability, explanatory power, and/or completeness
3. Determining the changes that would improve the validity, generalizability, completeness, and/or precision of a mathematical explanation
4. Evaluating a student conjecture for its validity and/or generalizability on a given domain
5. Evaluating mathematical definitions or other mathematical language for precision, validity, generalizability, usefulness in a particular context, and/or support for an instructional goal
Problems, Examples, and Structure

6. Evaluating mathematical problems for how well they elicit a particular idea, support the use of a particular solution strategy or practice, fit a particular mathematical structure, address the same concept as another problem, or assess a particular student conception or error
7. Writing mathematical problems that fit a particular solution strategy or mathematical structure
8. Evaluating examples for how well they introduce a concept; illustrate an idea or relationship; illustrate the appropriateness of a strategy, procedure, or practice; or address particular student questions, misconceptions, or partial conceptions
9. Generating or identifying nonexamples or counterexamples to highlight a mathematical distinction or to demonstrate why a student conjecture is incorrect or partially incorrect
10. Choosing which mathematical topics are most closely related to a particular instructional goal

Representations and Manipulatives

11. Selecting, creating, or evaluating representations or manipulatives for a mathematical purpose or to show a particular mathematical idea
12. Evaluating how representations or manipulatives have been used to show particular mathematical ideas, relationships between ideas, mathematical processes, or strategies in a text, talk, or written work

Student Strategies and Errors

13. Determining whether student work demonstrates the use of a particular mathematical idea or strategy
14. Determining whether a strategy is mathematically valid or generalizable
15. Interpreting a student’s mathematical error, including anticipating how it would replicate across similar problems, and choosing other work samples that demonstrate the same error
16. Identifying tasks or situations in which student work or talk that seems mathematically valid might mask incorrect thinking

Content Topics

This list details the mathematics topics critical for elementary students to master.

I. Counting and Operations with Whole Numbers

A. Counting
1. Counts and skip counts whole numbers between 0 and 1,000
2. Counts on, starting with any whole number
3. Connects counting to cardinality
4. Demonstrates understanding of one-to-one correspondence between numbers and objects being counted
5. Subitizes (recognizes small quantities by sight)
6. Identifies relationships between counting and the concept of larger and smaller numbers (i.e., that sets with higher counts are larger than sets with smaller counts)

B. Operations with Whole Numbers
1. Demonstrates understanding of representations of addition, subtraction, multiplication, and division (including objects such as manipulatives, drawings, and diagrams) and relates these representations of operations to expressions and equations
2. Solves mathematical and real-world problems involving the four operations, including solving problems by using properties of operations

II. Place Value and Decimals

1. Demonstrates a conceptual understanding of the value of the digits in a number
2. Compares multidigit and decimal numbers
3. Rounds multidigit and decimal numbers
4. Composes and decomposes multidigit numbers into groupings and understands why grouping and ungrouping are helpful in performing operations on multidigit and decimal numbers
5. Uses drawings and objects such as manipulatives to represent place value, relating these drawings and objects to numerical equations and written descriptions
III. Fractions, Operations with Fractions, and Ratios

1. Demonstrates understanding of fractions as part-whole relationships, as multiples of unit fractions, as numbers, and as ratios, moving back and forth flexibly among these conceptualizations
2. Demonstrates understanding of characteristics of fractions that are less than one, equal to one, and greater than one
3. Demonstrates understanding of equipartitioning and that it is a building block for understanding fractions as part-whole relationships
4. Demonstrates understanding of fraction equivalence
5. Uses a variety of strategies for comparing fractions
6. Performs operations such as addition, subtraction, multiplication, and division with fractions as well as with fractions and whole numbers, understanding and using different strategies for these operations and building intuition about how the operations work (e.g., recognizing that multiplying a whole number by a fraction that is less than one makes the product smaller)
7. Demonstrates understanding of applications of operations on fractions (e.g., scaling)

IV. Early Equations and Expressions, Measurement, and Geometry

A. Early Equations and Expressions

1. Demonstrates understanding of what it means for algebraic terms, expressions, and equations to be considered equivalent, how the equal sign is used to represent relational equivalence, and that equations maintain their equivalence status under certain algebraic manipulations
2. Determines whether equations are true, identifies the missing values that would make them true, solves equations using the four operations, and solves relational statements by substitution
3. Follows the standard order of operations (including the use of parentheses and the distributive property of multiplication over addition)

4. Demonstrates awareness of different interpretations of the word “variable,” including the ideas of quantities that are unknown (which underlies understanding how to solve equations) and quantities that vary (which can be connected to patterns and will support later understanding of functional relationships)
5. Uses the less-than and greater-than relational symbols (<, >) to compare quantities

B. Measurement

1. Describes measurable attributes of objects
2. Compares two objects with a common measurable attribute
3. Chooses appropriate measurement tools and uses the tools to take measurements
4. Calculates and estimates perimeter, area, volume, and measurements of angles in mathematical and real-world problems
5. Converts between measurement units

C. Geometry

1. Demonstrates understanding of shapes and their attributes
2. Composes and decomposes shapes
3. Draws shapes based on specific attributes such as number of angles and number of equal faces
4. Demonstrates understanding of lines, line segments, rays, and angles in two-dimensional figures
5. Classifies two-dimensional figures based on properties
Measuring Content Knowledge in Practice

The sample test question below demonstrates the relationship between the tasks of teaching mathematics and the content topics and subtopics in this test. Most questions on the test will measure the specialized content knowledge needed about a particular subtopic (e.g., I.A., Counting) to carry out a particular task of teaching. While all topics will be covered in the proportions listed in the chart on page 50 (30 percent for Counting and Operations with Whole Numbers; 25 percent for Place Value and Decimals; 25 percent for Fractions, Operations with Fractions, and Ratios; and 20 percent for Early Equations and Expressions, Measurement, and Geometry), the proportion of different tasks of teaching may vary.

\[
\begin{array}{ccc}
385 & 453 & 321 \\
+ 462 & + 427 & + 836 \\
7147 & 8710 & 1157 \\
\end{array}
\]

Josh is a third-grade student in Ms. Carter’s classroom. Josh’s answers to three addition problems are shown. He incorrectly answered the first two problems but correctly answered the third problem.

If Josh uses the same strategy to answer the following problem, what will his answer be?

\[
\begin{array}{c}
328 \\
+564 \\
\hline
\end{array}
\]

The question measures specialized content knowledge needed to teach operations with whole numbers; specifically, it deals with knowledge needed to interpret the error of a student learning to solve multidigit addition problems.

For the mathematics test, this question is classified as measuring content knowledge related to the topic of operations with whole numbers, in the context of Task of Teaching #15, “Interpreting a student’s mathematical error, including anticipating how it would replicate across similar problems, and choosing other work samples that demonstrate the same error.”

For the answer to this question, an explanation of the answer, and additional sample questions with explanatory information, please see Chapter 3. The sample questions in Chapter 3 demonstrate ways to measure content knowledge related to many different topics and tasks of teaching.
2. Familiarize Yourself with Test Questions

*Become comfortable with the types of questions you’ll find on the Praxis tests*

The Praxis Elementary Education: Content Knowledge for Teaching assessment is delivered on a computer. It includes a variety of question types, each intended to measure your ability to make different types of judgments similar to those made by elementary teachers. Most are one-point selected-response questions, for which you select one or more answers from a list of choices or make another kind of selection (e.g., by clicking on a sentence in a text or by clicking on part of a graphic). Numeric-entry questions, for which you enter a numeric value in an answer field, are also worth one point. The assessment also may include a small number of two-point questions, for which partial credit is available.

One-point questions may ask you to respond in a variety of ways, including the following.

- **Clicking an oval to select a single answer from a list of choices.** This is the most common question format.
- **Typing a number in an entry box.** When the answer is a number, you may be asked to enter a numerical response. Some questions may have more than one place to enter a response.
- **Clicking check boxes.** You may be asked to click check boxes instead of an oval when more than one choice within a set of answers can be selected. Sometimes the question will specify the number of intended responses. Other times you will be instructed to “Select all that apply”; for these questions, the number of correct answers can be anywhere from one to all of the answer choices given.
- **Clicking parts of a graphic.** In some questions, you will select your answers by clicking on a location (or locations) on a graphic, such as a map or chart, as opposed to choosing your answer from a list.
- **Clicking on text.** In questions with reading passages, you may be asked to choose your answers by clicking on a word, phrase, sentence, paragraph, or other section within the reading passage. Sometimes you will be asked to make more than one selection.
- **Dragging and dropping answer choices into targets on the screen.** You may be asked to select answers from a list of choices and drag your answers to the appropriate location in a table, paragraph of text, or graphic.
- **Selecting an answer choice from a drop-down menu.** You may be asked to choose an answer by selecting from a drop-down menu (e.g., to complete a sentence).

Two-point questions, for which partial credit is available, may ask you to respond in one of the following two ways.

- **Selecting multiple answers for composite (two-part) questions.** For this type of question, you will click on your answer(s) to part A and then click on your answer(s) to part B. Because each part of the composite question may have different directions, make sure to read the directions carefully for both part A and part B. If you answer part A correctly but do not provide the correct answer(s) for part B, you will earn one point. You cannot receive credit for part B unless part A is answered correctly.
- **Selecting boxes in a table.** In some questions, answer choices will appear in a table with three or more rows. You will be asked to select one choice for each row. You will receive full credit (two points) if all rows are completed correctly. You will receive one point if all but one row is completed correctly. You will receive zero points if two or more rows are not completed correctly.

You may be familiar with these question formats from taking other standardized tests. If not, familiarize yourself with them so you don’t spend time during the test figuring out how to answer them.

Perhaps the best way to understand computer-delivered questions is to view the Computer-Delivered Testing Demonstration on the Praxis web site to learn how a computer-delivered test works and see examples of some types of questions you may encounter.

**Understanding Selected-Response Questions**
Many selected-response questions begin with the phrase “which of the following.” Take a look at this example:

Which of the following is a flavor made from beans?
(A) Strawberry
(B) Cherry
(C) Vanilla
(D) Mint

How would you answer this question?
All of the answer choices are flavors. Your job is to decide which of the flavors is the one made from beans.

Try following these steps to select the correct answer.

1) Limit your answer to the choices given. You may know that chocolate and coffee are also flavors made from beans, but they are not listed. Rather than thinking of other possible answers, focus only on the choices given (“which of the following”).

2) Eliminate incorrect answers. You may know that strawberry and cherry flavors are made from fruit and that mint flavor is made from a plant. That leaves vanilla as the only possible answer.

3) Verify your answer. You can substitute “vanilla” for the phrase “which of the following” and turn the question into this statement: “Vanilla is a flavor made from beans.” This will help you be sure that your answer is correct. If you’re still uncertain, try substituting the other choices to see if they make sense. You may want to use this technique as you answer selected-response questions on the practice tests.

Try a more challenging example
The vanilla bean question is pretty straightforward, but you’ll find that more challenging questions have a similar structure. For example:

Entries in outlines are generally arranged according to which of the following relationships of ideas?
(A) Literal and inferential
(B) Concrete and abstract
(C) Linear and recursive
(D) Main and subordinate

You’ll notice that this example also contains the phrase “which of the following.” This phrase helps you determine that your answer will be a “relationship of ideas” from the choices provided. You are supposed to find the choice that describes how entries, or ideas, in outlines are related.

Sometimes it helps to put the question in your own words. Here, you could paraphrase the question in this way: “How are outlines usually organized?” Since the ideas in outlines usually appear as main ideas and subordinate ideas, the answer is (D).
QUICK TIP: Don’t be intimidated by words you may not understand. It might be easy to be thrown by words like “recursive” or “inferential.” Read carefully to understand the question and look for an answer that fits. An outline is something you are probably familiar with and expect to teach to your students. So slow down, and use what you know.

Watch out for selected-response questions containing “NOT,” “LEAST,” and “EXCEPT”
This type of question asks you to select the choice that does not fit. You must be very careful because it is easy to forget that you are selecting the negative. This question type is used in situations in which there are several good solutions or ways to approach something, but also a clearly wrong way.

How to approach questions about graphs, tables, or reading passages
When answering questions about graphs, tables, or reading passages, provide only the information that the questions ask for. In the case of a map or graph, you might want to read the questions first, and then look at the map or graph. In the case of a long reading passage, you might want to go ahead and read the passage first, noting places you think are important, and then answer the questions. Again, the important thing is to be sure you answer the questions as they refer to the material presented. So read the questions carefully.

How to approach unfamiliar formats
New question formats are developed from time to time to find new ways of assessing knowledge. Tests may include audio and video components, such as a movie clip or animation, instead of a map or reading passage. Other tests may allow you to zoom in on details in a graphic or picture.

Tests may also include interactive questions. These questions take advantage of technology to assess knowledge and skills in ways that standard selected-response questions cannot. If you see a format you are not familiar with, read the directions carefully. The directions always give clear instructions on how you are expected to respond.

QUICK TIP: Don’t make the questions more difficult than they are. Don’t read for hidden meanings or tricks. There are no trick questions on Praxis tests. They are intended to be serious, straightforward tests of your knowledge.
3. Practice with Sample Test Questions

Answer practice questions and find explanations for correct answers

Sample Test Questions

The sample questions that follow are examples of the kinds of questions that are on the test. They are not, however, representative of the entire scope of the test in either content or difficulty. Answers with explanations follow the questions, along with information about the content topic, subtopic, and task of teaching targeted in each question.

The sample questions that follow are examples of the kinds of questions that are on the test. They are not, however, representative of the entire scope of the test in either content or difficulty. Answers with explanations follow the questions, along with information about the content topic, subtopic, and task of teaching targeted in each question.

Directions: Select the best answer or answers for each question below. Questions in formats that may be unfamiliar are followed by a note entitled “How to Answer the Question Above.”

1. Which three of the following expressions are equivalent to $3,966 \times 4$?
   
   (A) $3,000 \times 4 + 900 \times 4 + 50 \times 4 + 6 \times 4$
   
   (B) $(4,000 \times 4 - 100 \times 4) + (60 \times 4 - 4 \times 4)$
   
   (C) $4 \times 3 + 4 \times 9 + 4 \times 5 + 4 \times 6$
   
   (D) $4,000 \times 4 - 40 \times 4 - 4 \times 4$
   
   (E) $3 \times 1,000 \times 4 + 95 \times 100 \times 4 + 6 \times 1 \times 4$

2. Which of the following word problems can be answered by finding the quotient of $\frac{3}{4}$ and $\frac{1}{3}$?

   Select all that apply.

   (A) Casey poured $\frac{3}{4}$ quarts of fruit punch into cups. She filled each cup with $\frac{1}{3}$ quart of fruit punch. How many cups did Casey fill?

   (B) A pump working at a constant rate filled $\frac{3}{4}$ equal-sized tanks of water in $\frac{1}{3}$ hour. At the same rate, how many tanks will the pump fill in 1 hour?

   (C) Laura uses $\frac{1}{3}$ of a piece of ribbon that is $\frac{3}{4}$ feet long to wrap a present. What is the length of the ribbon she used to wrap the present?

How to Answer the Question Above

This is a “select all that apply” question. You should select one, two, or all three of the answer choices—however many are correct. You earn credit for the question only if you select all of the choices that are correct answers. In the actual test, the answer choices appear next to empty check boxes. Click on a box to select the choice next to it; this causes an “x” to appear in the box. If you change your mind, click the box again to remove the “x.”
3. Dora made a pile of 5 counters. Then Mr. Levy asked her to add counters to her pile of 5 so that the pile would have 7 counters. Dora counted out 7 more counters and added them to the pile of 5 counters.

Which of the following most likely explains the reason behind Dora’s error?

(A) Dora does not fully understand one-to-one correspondence between numbers and objects.

(B) Dora does not yet have a concept of the quantity 7.

(C) Dora does not yet understand that one quantity can be composed of two smaller quantities.

(D) Dora does not yet know her number facts for sums greater than 10.

4. 

\[
\begin{array}{ccc}
385 & 453 & 321 \\
+462 & +427 & +836 \\
7147 & 8710 & 1157
\end{array}
\]

Josh is a third-grade student in Ms. Carter’s classroom. Josh’s answers to three addition problems are shown. He incorrectly answered the first two problems but correctly answered the third problem.

If Josh uses the same strategy to answer the following problem, what will his answer be?

\[
\begin{array}{c}
328 \\
+564
\end{array}
\]

5. Mr. Keller’s sixth-grade class is learning about algebraic equations. In his teachers’ edition of the textbook, Mr. Keller finds a page that suggests he ask students to critique the following two solutions to determine whether they are valid.

\[
\begin{array}{c|c}
4x + 2 = 12 & 5 = 2x + 3 \\
6x = 12 & 5 = 5x \\
x = 12 \div 6 & x = 2 \\
x = 2 & 5 + 5 = x \\
x = 2 & x = 1
\end{array}
\]

Which of the following is most clearly highlighted by asking students to critique the invalid strategies?

(A) Understanding the meaning of the equal sign

(B) Understanding the importance of combining like terms

(C) Understanding the use of properties of operations to simplify expressions

(D) Understanding the use of inverse operations to solve equations
6. Ms. Dale wants her students to develop mental strategies that can be used to find the answer to addition and subtraction problems, including composing and decomposing numbers based on place value.

In one lesson, she asks her students to find numbers whose sum or difference is 28. She then has seven students share their answers as she writes them on the board.

Which three of the following student answers are most closely related to Ms. Dale’s goal that students will be able to compose and decompose numbers based on place value?

(A) $7 + 7 + 7 + 7$
(B) $8 + 10 + 10$
(C) $14 + 14$
(D) $20 + 8$
(E) $20 + 10 - 2$
(F) $25 + 3$
(G) $39 - 11$

How to Answer the Question Above
This is a multiple-choice question with three correct answers. You must select all three correct answers—and no incorrect answers—to earn credit for the question. In the actual test, the answer choices appear next to empty check boxes. Click on a box to select the answer choice next to it; this causes an “x” to appear in the box. If you change your mind, click the box again to remove the “x.”

7. 

$$6 + 2 = 3 \quad 3 \times 2 = 6$$

$$21 + 7 = 3 \quad 3 \times 7 = 21$$

Mr. Khan’s students are discussing the problems shown. Mr. Khan asks his students what relationships they notice in the problems. One student responds with the following conjecture.

I noticed that when you divide by a number and then multiply the result by the same number, you always get back the first number.

Provided that division by zero is excluded, for which of the following sets of numbers is the student’s conjecture true?

Select all that apply.

(A) Whole numbers
(B) Integers
(C) Fractions and decimals

How to Answer the Question Above
This is a “select all that apply” question. You should select one, two, or all three of the answer choices—however many are correct. You earn credit for the question only if you select all of the choices that are correct answers. In the actual test, the answer choices appear next to empty check boxes. Click on a box to select the choice next to it; this causes an “x” to appear in the box. If you change your mind, click the box again to remove the “x.”
8. A student found an incorrect answer to the problem \( \frac{3}{4} \div \frac{5}{6} \). The student's answer is represented in the work shown.

\[
\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{19}{24}
\]

Which of the following student work samples shows work that is most similar to the preceding work?

- (A) \( \frac{3}{8} + \frac{2}{3} = \frac{3}{24} + \frac{2}{24} = \frac{5}{24} \)
- (B) \( \frac{4}{5} + \frac{1}{2} = \frac{16}{20} + \frac{10}{20} = \frac{26}{20} \)
- (C) \( \frac{5}{7} + \frac{3}{4} = \frac{9}{22} + \frac{10}{22} = \frac{19}{22} \)
- (D) \( \frac{1}{2} + \frac{7}{9} = \frac{9}{18} + \frac{14}{18} = \frac{23}{36} \)

9. Rosana had a total of 9 shirts. She gave 2 to Emily. How many shirts does Rosana have now?

Which of the following problems has the same mathematical structure as the problem above?

- (A) Rosana used 7 paint colors for her project. Emily used 2 different paint colors for her project. How many paint colors did Rosana and Emily use together?
- (B) Rosana has some books. She bought 1 more book. Now she has 8 books. How many books did Rosana start with?
- (C) Rosana has a total of 3 stickers. Emily has 6 more stickers than Rosana. How many stickers does Emily have?
- (D) Rosana brought 5 cookies for lunch. How many cookies did she have after she ate 4 of the cookies?

10. Ms. Hayes asked her students to calculate the difference \( 0.7 - 0.07 \) by converting the decimals into base-ten fractions.

One student, Daryl, answered the problem as represented in the work shown.

\[
0.7 - 0.07 = \frac{7}{10} - \frac{7}{100} = \frac{70}{100} - \frac{7}{100} = \frac{63}{100} = 0.63
\]

When Ms. Hayes asked Daryl to explain his strategy, he said, “The answer is 63 hundredths. I wrote the decimals 7 tenths and 7 hundredths as fractions and subtracted them. Since I wanted the denominators to be the same, I added a zero to the first 7 and a zero to 10. 70 hundredths minus 7 hundredths is 63 hundredths.”

Which of the following changes to Daryl’s explanation is best for clarifying the mathematics that underlie his strategy?

- (A) He should indicate why \( 0.7 = \frac{7}{10} \) and \( 0.07 = \frac{7}{100} \). 
- (B) He should point out that \( \frac{7\times10}{10\times10} = \frac{70}{100} \). 
- (C) He should point out that \( \frac{70}{100} - \frac{7}{100} = \frac{70-7}{100} \). 
- (D) He should indicate why \( 0.63 = \frac{63}{100} \).
11. Mr. Bass is working on defining quadrilaterals with his students. He notices that many students are focused on the number of sides, saying things like “a quadrilateral is a shape with four sides.”

Which two of the following figures are most likely to support students in refining their definition of quadrilaterals?

(A)  
(B)  
(C)  
(D)  

How to Answer the Question Above
This is a multiple-choice question with two correct answers. You must select both correct answers—and no incorrect answers—to earn credit for the question. In the actual test, the answer choices appear next to empty check boxes. Click on a box to select the answer choice next to it; this causes an “x” to appear in the box. If you change your mind, click the box again to remove the “x.”

12. Ms. Howe’s students are learning how to use models to help them answer word problems. The models use bars to represent the relationships between the given quantities and the unknown quantity. In each model, the unknown quantity is represented with a question mark. The quantities given in the word problem will occupy the other boxes.

Ms. Howe presents the following word problem to her students.

Max had $24. He gave \( \frac{1}{3} \) of his money to Sarah and the rest to Olivia. How much money did he give to Olivia?

Which of the following models best corresponds to the given word problem?

(A)  
(B)  
(C)  
(D)
13. Ms. Kress asked her students to compare $\frac{1}{3}$ and $\frac{7}{8}$. Four of her students correctly answered that $\frac{7}{8}$ is greater than $\frac{1}{3}$, but they gave different explanations when asked to describe their strategies to the class.

Indicate whether each of the following student explanations provides evidence of a mathematically valid strategy for comparing $\frac{1}{3}$ and $\frac{7}{8}$.

<table>
<thead>
<tr>
<th>Student Explanation</th>
<th>Provides Evidence</th>
<th>Does Not Provide Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>When you look at the numbers, you see that 7 is bigger than 1, so $\frac{7}{8}$ is the bigger fraction.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>In the first fraction, 1 is less than half of 3, but in the second, 7 is more than half of 8, so $\frac{7}{8}$ is larger than $\frac{1}{3}$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I multiplied 1 times 7 and 3 times 7, so $\frac{1}{3}$ is the same as $\frac{7}{21}$. This means that $\frac{7}{8}$ is bigger than $\frac{1}{3}$ because $\frac{1}{8}$ is bigger than $\frac{1}{21}$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I wanted to make a fraction equal to $\frac{1}{3}$ with the same bottom number as $\frac{7}{8}$, so I added 5 to 3 and got 8. Then I added 5 to 1 and got 6, but 7 is greater than 6, so $\frac{7}{8}$ is greater.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**How to Answer the Question Above**

This is a table question worth two points and eligible for partial credit. It requires you to select one choice for each row. You will receive full credit (two points) if all rows are completed correctly. You will receive one point if all but one row is completed correctly. In the actual test, click on a box to select it; a check mark will appear. If you change your mind, click on the check mark to remove it, or simply click on another box in the same row, and your check mark will move to the new box.
Mathematics—CKT Answers

The answers to the sample test questions are provided below, along with explanations and classifications. Each question focuses on a specific mathematics topic and subtopic listed in “Content Topics,” beginning on page 7. Questions that measure specialized content knowledge also incorporate a particular “Task of Teaching Mathematics” listed on page 6. These questions are intended to measure the specialized content knowledge needed to carry out the task effectively.

1. The correct answers are (A), (B), and (D). Since 3,956 can be written as $3,000 + 900 + 50 + 6$, the given expression is equivalent to $(3,000 + 900 + 50 + 6) \times 4$. Applying the distributive property yields $3,000 \times 4 + 900 \times 4 + 50 \times 4 + 6 \times 4$, which is the expression in (A). Since 3,956 can be written as $3,900 + 56$, the given expression is equivalent to $(3,900 + 56) \times 4$. Applying the distributive property yields $3,900 \times 4 + 56 \times 4$. One can rewrite 3,900 as $4,000 - 100$ and 56 as $60 - 4$, which yields the equivalent expression $(4,000 - 100) \times 4 + (60 - 4) \times 4$. Applying the distributive property again yields $(4,000 - 100) \times 4 + (60 - 4) \times 4$, which is the expression in (B). Since 3,956 can be written as $4,000 - 40 - 4$, the given expression is equivalent to $(4,000 - 40 - 4) \times 4$. Applying the distributive property yields $4,000 \times 4 - 40 \times 4 - 4 \times 4$, which is the expression in (D). Applying the distributive property to the expression in (C) yields $4 \times (3 + 9 + 5 + 6)$, which is equivalent to $4 \times 23$, but this expression is not equivalent to the given expression. Since $6 \times 1 = 6$, applying the distributive property to the expression in (E) yields $(3 \times 1000 + 95 \times 100 + 6) \times 4$, which is equivalent to $(3,000 + 950 + 6) \times 4$. The sum of the numbers in the parentheses is 12,506; therefore, the expression in (E) is not equivalent to the given expression.

<table>
<thead>
<tr>
<th>Content Knowledge Type</th>
<th>Work of the Student Curriculum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Task of Teaching Mathematics</td>
<td>not applicable</td>
</tr>
<tr>
<td>Topic</td>
<td>II. Place Value and Decimals</td>
</tr>
</tbody>
</table>

2. The correct answers are (A) and (B). The problem in (A) is a measurement division problem. Solving the problem involves answering the question, “How many $\frac{1}{3}$-quart units are there in $\frac{3}{4}$ quarts?” The answer can be found by dividing $\frac{3}{4}$ by $\frac{1}{3}$. The problem in (B) is a unit rate problem, since it asks how many tanks the pump will fill in 1 hour. Solving the problem involves answering the question, “What is the rate of tanks per hour at which the pump is working if it fills $\frac{3}{4}$ tanks in $\frac{1}{3}$ hour?” Since the rate is measured in tanks per hour, the answer can be found by dividing $\frac{3}{4}$ by $\frac{1}{3}$. Solving the problem in (C) involves answering the question, “What is $\frac{1}{3}$ of $\frac{3}{4}$ feet?” The answer can be found by multiplying $\frac{3}{4}$ by $\frac{1}{3}$ but cannot be found by dividing $\frac{3}{4}$ by $\frac{1}{3}$, so (C) is not correct.
3. The correct answer is (C). Dora counted out 7 more counters, not realizing that 5 can be part of 7, so she does not seem to understand that one quantity can be composed of two smaller quantities. (A) is not the key because Dora actually counted out 7 more counters, so there is evidence that she does understand one-to-one correspondence. (B) is not the key because Dora counted out 7 counters, so there is evidence that she has a concept of the quantity 7. (D) is not the key because even though Dora made a pile of 12 counters, knowing number facts for sums greater than 10 was not necessary for the original task, which was to add counters to her pile of 5 counters so there would be 7 counters in the pile. Therefore, (D) does not explain the reason behind Dora’s error.

<table>
<thead>
<tr>
<th>Content Knowledge Type</th>
<th>Specialized Content Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Task of Teaching Mathematics</td>
<td>15. Interpreting a student’s mathematical error, including anticipating how it would replicate across similar problems, and choosing other work samples that demonstrate the same error</td>
</tr>
<tr>
<td>Topic</td>
<td>I. Counting and Operations with Whole Numbers</td>
</tr>
<tr>
<td>Subtopic</td>
<td>A. Counting</td>
</tr>
</tbody>
</table>

4. The correct answer is 8812. Josh's error is that he is not regrouping when necessary; instead, he is just writing the sum of the digits in each place value column. His written answer is correct in the third problem because 11 hundreds (the result of adding 3 hundreds and 8 hundreds) is equivalent to regrouping to get 1100. However, when he does not regroup in the first two problems, his written answers are incorrect. For example, in the first problem, Josh adds 8 tens and 6 tens to get 14 tens, but instead of regrouping 10 of those tens to get 100 and then writing the final answer as 847, Josh just adds the 3 hundreds and the 4 hundreds and then writes the final answer as 7147. Therefore, if Josh uses the same method in the last problem, he will add 8 and 4 to get 12 ones, but he will not regroup, and then he will add 2 and 6 to get 8 and 3 and 5 to get 8, and his final answer will be 8812.

<table>
<thead>
<tr>
<th>Content Knowledge Type</th>
<th>Specialized Content Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Task of Teaching Mathematics</td>
<td>8. Evaluating examples for how well they introduce a concept; illustrate an idea or relationship; illustrate the appropriateness of a strategy, procedure or practice; or address particular student questions, misconceptions, or partial conceptions</td>
</tr>
<tr>
<td>Topic</td>
<td>IV. Early Equations and Expressions, Measurement, and Geometry</td>
</tr>
<tr>
<td>Subtopic</td>
<td>A. Early Equations and Expressions</td>
</tr>
</tbody>
</table>
6. The correct answers are (B), (D), and (E). A decomposition of numbers based on place value means that the number is written as the sum or difference of tens and ones. In (B), the number 28 is decomposed as the sum of 2 tens, 10 + 10, and 8 ones. In (D), the number 28 is decomposed as the sum of 2 tens and 8 ones. In (E), the number 28 is decomposed as the difference of 3 tens, written as the sum of 2 tens and 1 ten, and 2 ones. (A) is incorrect because the number 28 is decomposed as a repeated addition of the number 7, so this decomposition is related to skip counting. (C) is incorrect because the number 28 is decomposed as the sum of 14 and itself, so this decomposition is related to doubles. (F) and (G) are incorrect because although both answers are decompositions of 28, neither of the numbers in either decomposition is a multiple of 10.

7. The correct answers are (A), (B), and (C). The student’s conjecture can be represented by the equation \( (a + b) \times b = a \), where \( a \) and \( b \) are numbers. Based on the order of operations, the left-hand side of the equation can be written as \( a + b \times b \), and since multiplication and division by the same number are inverse operations, it is true in general that \( (a + b) \times b = a \), with one exception. The exception is when \( b = 0 \), since division by zero is undefined. However, the question excludes division by zero, so the student’s conjecture is true for whole numbers, integers, and fractions and decimals. Whole numbers are the numbers \( 0, 1, 2, 3, \ldots \), and integers are positive and negative whole numbers.

8. The correct answer is (D). The student answered the problem \( \frac{3}{4} + \frac{5}{6} \). The student first correctly found equivalent fractions to \( \frac{3}{4} \) and \( \frac{5}{6} \) that shared the least common denominator. However, the student then incorrectly added the fractions by adding the denominators to get an answer of \( \frac{19}{24} \) rather than using the common denominator, which would have led to the correct answer of \( \frac{19}{12} \). Similarly, in (D), the work shows that the student correctly found equivalent fractions that shared the least common denominator, but the student then incorrectly added the fractions by adding the denominators. The work in (A) is different because the student finds the least common denominator, but the rewritten fractions are not equivalent to the original fractions. The work in (B) is different because the student does not find the least common denominator, and then the student correctly adds the fractions. The work in (C) is different because the student did not correctly find equivalent fractions that shared the least common denominator.
9. The correct answer is (D). In the original problem, an initial quantity is given, a portion of it is taken away, and the problem asks for the resulting quantity. (D) has the same structure. In the problem in (A), two parts are given (the paint colors Rosana used and the paint colors that Emily used), and the problem asks for the whole. In the problem in (B), the initial quantity is unknown, an additional book is joined to that quantity, and the end result is given. In the problem in (C), an initial quantity is given (Rosana’s stickers), the amount by which it differs from a second quantity (Emily’s stickers) is also given, and the problem asks for the size of the second quantity (Emily’s stickers). While this problem, like the original one, can be solved directly by subtraction, the structure reflects a comparison rather than a reduction of quantity, and the information is presented in a different order.

<table>
<thead>
<tr>
<th>Content Knowledge Type</th>
<th>Task of Teaching Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specialized Content Knowledge</td>
<td>6. Evaluating mathematical problems for how well they elicit a particular idea, support the use of a particular solution strategy or practice, fit a particular mathematical structure, address the same concept as another problem, or assess a particular student conception or error</td>
</tr>
<tr>
<td>Topic</td>
<td>I. Counting and Operations with Whole Numbers</td>
</tr>
<tr>
<td>Subtopic</td>
<td>B. Operations with Whole Numbers</td>
</tr>
</tbody>
</table>

10. The correct answer is (B). When Daryl tried to explain why \( \frac{7}{10} = \frac{70}{100} \), what he really said was that \( \frac{7 + 0}{10 + 0} = \frac{70}{100} \). However, 70 comes from multiplying 7 by 10, and 100 comes from multiplying 10 by 10, so a better explanation would be to say that he had to multiply the numerator and denominator of \( \frac{7}{10} \) by 10 to obtain the equivalent fraction \( \frac{70}{100} \). (A) is incorrect because Daryl clearly conveyed that \( 0.7 = \frac{7}{10} \) and \( 0.07 = \frac{7}{100} \) by referring to the decimals 0.7 and 0.07 as 7 tenths and 7 hundredths, respectively, which linked each of the two decimal numbers to its corresponding base-ten fraction. (C) is incorrect because Daryl did point out that \( \frac{70}{100} - \frac{7}{10} = \frac{70 - 7}{100} \) when he said, “70 hundredths minus 7 hundredths is 63 hundredths.” (D) is incorrect because Daryl referred to the decimal 0.63 as 63 hundredths, which was a direct link to the base-ten fraction \( \frac{63}{100} \).

<table>
<thead>
<tr>
<th>Content Knowledge Type</th>
<th>Task of Teaching Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specialized Content Knowledge</td>
<td>3. Determining the changes that would improve the validity, generalizability, completeness, and/or precision of a mathematical explanation</td>
</tr>
<tr>
<td>Topic</td>
<td>II. Place Value and Decimals</td>
</tr>
</tbody>
</table>
11. The correct answers are (A) and (C). The figure in (A) has four sides, but one side is curved, which highlights the need to clarify that the sides of quadrilaterals must be line segments. The figure in (C) has four sides but is not closed, which highlights the need to clarify that quadrilaterals are closed figures. (B) is not a correct choice because the students would correctly say the figure is not a quadrilateral since it does not have four sides, so it would not support the students in refining their definition of quadrilaterals. (D) is not a correct choice because, based on their definition, the students would correctly say that it is a quadrilateral, so it would not support the students in refining their definition of quadrilaterals.

12. The correct answer is (C). First, since it is known that Max had $24, the model should show that the total amount is known. Next, since Max gave 1/3 of his money to Sarah, the model should show that the total is divided in thirds. Finally, the model should show that the money given to Olivia, which is 2/3 of the total amount, is the unknown. (C) is the only choice that shows that the total amount would be filled in, the total amount is divided in thirds, and the unknown is 2/3 of the total amount. (A) is incorrect because it shows that the unknown is the total amount, which is not true. (B) is incorrect because it shows that the unknown is the smaller part of the whole, which is incorrect. Also, note that in both (A) and (B), all that is given is that one part is larger than the other, not that one part is 2/3 of the whole and the other part is 1/3 of the whole. (D) is incorrect because it shows that the unknown is the total amount, which is not true.
13. The first and fourth explanations do not provide evidence of a mathematically valid strategy for comparing $\frac{1}{3}$ and $\frac{7}{8}$, but the second and third explanations do. In the first explanation, the student only compares the numerators of the fractions, which is not a valid strategy because it does not take into account the effect of the denominator on the size of the pieces. In the second explanation, the student compares both fractions to the benchmark fraction $\frac{1}{2}$, which is a valid strategy since $\frac{1}{3}$ is less than $\frac{1}{2}$ and $\frac{7}{8}$ is greater than $\frac{1}{2}$. In the third explanation, the student uses multiplicative reasoning to find a common numerator, and then the student compares the fractions by reasoning about the sizes of the unit fractions $\frac{1}{8}$ and $\frac{1}{21}$. This is a valid strategy. In the fourth explanation, the student uses additive reasoning to try to find a fraction equivalent to $\frac{1}{3}$ that has a denominator of 8, but $\frac{6}{8}$ is not equivalent to $\frac{1}{3}$, so this strategy is not valid.

<table>
<thead>
<tr>
<th>Content Knowledge Type</th>
<th>Specialized Content Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Task of Teaching Mathematics</td>
<td>2. Evaluating mathematical explanations for their validity, generalizability, explanatory power, and/or completeness</td>
</tr>
<tr>
<td>Topic</td>
<td>III. Fractions, Operations with Fractions, and Ratios</td>
</tr>
</tbody>
</table>
4. Determine Your Strategy for Success

Set clear goals and deadlines so your test preparation is focused and efficient

Effective Praxis test preparation doesn’t just happen. You’ll want to set clear goals and deadlines for yourself along the way. Otherwise, you may not feel ready and confident on test day.

1) Learn what the test covers.

You may have heard that there are several different versions of the same test. It’s true. You may take one version of the test and your friend may take a different version a few months later. Each test has different questions covering the same subject area, but both versions of the test measure the same skills and content knowledge.

You’ll find specific information on the test you’re taking on page 5, which outlines the content categories that the test measures and what percentage of the test covers each topic. Visit www.ets.org/praxis/testprep for information on other Praxis tests.

2) Assess how well you know the content.

Research shows that test takers tend to overestimate their preparedness—this is why some test takers assume they did well and then find out they did not pass.

The Praxis tests are demanding enough to require serious review of likely content, and the longer you’ve been away from the content, the more preparation you will most likely need. If it has been longer than a few months since you’ve studied your content area, make a concerted effort to prepare.

3) Collect study materials.

Gathering and organizing your materials for review are critical steps in preparing for the Praxis tests. Consider the following reference sources as you plan your study:

• Did you take a course in which the content area was covered? If yes, do you still have your books or your notes?

• Does your local library have a high school-level textbook in this area? Does your college library have a good introductory college-level textbook in this area?

Practice materials are available for purchase for many Praxis tests at www.ets.org/praxis/testprep. Test preparation materials include sample questions and answers with explanations.

4) Plan and organize your time.

You can begin to plan and organize your time while you are still collecting materials. Allow yourself plenty of review time to avoid cramming new material at the end. Here are a few tips:

• Choose a test date far enough in the future to leave you plenty of preparation time. Test dates can be found at www.ets.org/praxis/institutions/scores/passing/.

• Work backward from that date to figure out how much time you will need for review.

• Set a realistic schedule—and stick to it.
Step 4: Determine Your Strategy for Success

5) Understand how questions will be scored.

Scoring information can be found on page 46.

6) Develop a study plan.

A study plan provides a road map to prepare for the Praxis tests. It can help you understand what skills and knowledge are covered on the test and where to focus your attention. Use the study plan template on page 30 to organize your efforts.

And most important—get started!

Would a Study Group Work for You?

Using this guide as part of a study group

People who have a lot of studying to do sometimes find it helpful to form a study group with others who are working toward the same goal. Study groups give members opportunities to ask questions and get detailed answers. In a group, some members usually have a better understanding of certain topics, while others in the group may be better at other topics. As members take turns explaining concepts to one another, everyone builds self-confidence.

If the group encounters a question that none of the members can answer well, the group can go to a teacher or other expert and get answers efficiently. Because study groups schedule regular meetings, members study in a more disciplined fashion. They also gain emotional support. The group should be large enough so that multiple people can contribute different kinds of knowledge, but small enough so that it stays focused. Often, three to six members is a good size.

Here are some ways to use this guide as part of a study group:

• Plan the group's study program. Parts of the study plan template, beginning on page 30 can help to structure your group's study program. By filling out the first five columns and sharing the worksheets, everyone will learn more about your group's mix of abilities and about the resources, such as textbooks, that members can share with the group. In the sixth column ("Dates I will study the content"), you can create an overall schedule for your group's study program.

• Plan individual group sessions. At the end of each session, the group should decide what specific topics will be covered at the next meeting and who will present each topic. Use the topic headings and subheadings in the Test at a Glance table on page 5 to select topics, and then select practice questions, beginning on page 13.

• Prepare your presentation for the group. When it's your turn to present, prepare something that is more than a lecture. Write two or three original questions to pose to the group. Practicing writing actual questions can help you better understand the topics covered on the test as well as the types of questions you will encounter on the test. It will also give other members of the group extra practice at answering questions.

• Take a practice test together. The idea of a practice test is to simulate an actual administration of the test, so scheduling a test session with the group will add to the realism and may also help boost everyone's confidence. Remember, complete the practice test using only the time that will be allotted for that test on your administration day.

• Learn from the results of the practice test. Review the results of the practice test, including the number of questions answered correctly in each content category. For tests that contain constructed-
response questions, look at the Sample Test Questions section, which also contain sample responses to those questions and shows how they were scored. Then try to follow the same guidelines that the test scorers use.

- **Be as critical as you can.** You’re not doing your study partner(s) any favors by letting them get away with an answer that does not cover all parts of the question adequately.

- **Be specific.** Write comments that are as detailed as the comments about the sample responses. Indicate where and how your study partner(s) are doing an inadequate job of answering the question. Writing notes in the margins of the answer sheet may also help.

- **Be supportive.** Include comments that point out what your study partner(s) got right.

Then plan one or more study sessions based on aspects of the questions on which group members performed poorly. For example, each group member might be responsible for rewriting one paragraph of a response in which someone else did an inadequate job.

Whether you decide to study alone or with a group, remember that the best way to prepare is to have an organized plan. The plan should set goals based on specific topics and skills that you need to learn, and it should commit you to a realistic set of deadlines for meeting those goals. Then you need to discipline yourself to stick with your plan and accomplish your goals on schedule.
### 5. Develop Your Study Plan

**Develop a personalized study plan and schedule**

Planning your study time is important because it will help ensure that you review all content areas covered on the test. Use the sample study plan below as a guide. It shows a plan for the Core Academic Skills for Educators: Reading test. Following that is a study plan template that you can fill out to create your own plan. Use the “Learn about Your Test” and “Test Specifications” information beginning on page 5 to help complete it.

Use this worksheet to:
1. Define Content Areas: List the most important content areas for your test as defined in chapter 1.
2. Determine Strengths and Weaknesses: Identify your strengths and weaknesses in each content area.
3. Identify Resources: Identify the books, courses, and other resources you plan to use for each content area.
4. Study: Create and commit to a schedule that provides for regular study periods.

**Praxis Test Name (Test Code):** Core Academic Skills for Educators: Reading (5712)
**Test Date:** 9/15/15

<table>
<thead>
<tr>
<th>Content covered</th>
<th>Description of content</th>
<th>How well do I know the content? (scale 1–5)</th>
<th>What resources do I have/need for the content?</th>
<th>Where can I find the resources I need?</th>
<th>Dates I will study the content</th>
<th>Date completed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Key Ideas and Details</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Close reading</td>
<td>Draw inferences and implications from the directly stated content of a reading selection</td>
<td>3</td>
<td>Middle school English textbook</td>
<td>College library, middle school teacher</td>
<td>7/15/15</td>
<td>7/15/15</td>
</tr>
<tr>
<td>Determining Ideas</td>
<td>Identify summaries or paraphrases of the main idea or primary purpose of a reading selection</td>
<td>3</td>
<td>Middle school English textbook</td>
<td>College library, middle school teacher</td>
<td>7/17/15</td>
<td>7/17/15</td>
</tr>
<tr>
<td>Determining Ideas</td>
<td>Identify summaries or paraphrases of the supporting ideas and specific details in a reading selection</td>
<td>3</td>
<td>Middle and high school English textbook</td>
<td>College library, middle and high school teachers</td>
<td>7/20/15</td>
<td>7/21/15</td>
</tr>
<tr>
<td>Craft, Structure, and Language Skills</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Interpreting tone</td>
<td>Determine the author’s attitude toward material discussed in a reading selection</td>
<td>4</td>
<td>Middle and high school English textbook</td>
<td>College library, middle and high school teachers</td>
<td>7/25/15</td>
<td>7/26/15</td>
</tr>
<tr>
<td>Analysis of structure</td>
<td>Identify key transition words and phrases in a reading selection and how they are used</td>
<td>3</td>
<td>Middle and high school English textbook, dictionary</td>
<td>College library, middle and high school teachers</td>
<td>7/25/15</td>
<td>7/27/15</td>
</tr>
<tr>
<td>Analysis of structure</td>
<td>Identify how a reading selection is organized in terms of cause/effect, compare/contrast, problem/solution, etc.</td>
<td>5</td>
<td>High school textbook, college course notes</td>
<td>College library, course notes, high school teacher, college professor</td>
<td>8/1/15</td>
<td>8/1/15</td>
</tr>
<tr>
<td>Author’s purpose</td>
<td>Determine the role that an idea, reference, or piece of information plays in an author’s discussion or argument</td>
<td>5</td>
<td>High school textbook, college course notes</td>
<td>College library, course notes, high school teacher, college professor</td>
<td>8/1/15</td>
<td>8/1/15</td>
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</tbody>
</table>

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## Step 5: Develop Your Study Plan

<table>
<thead>
<tr>
<th>Content covered</th>
<th>Description of content</th>
<th>How well do I know the content? (scale 1–5)</th>
<th>What resources do I have/need for the content?</th>
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<th>Dates I will study the content</th>
<th>Date completed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Language in different contexts</td>
<td>Determine whether information presented in a reading selection is presented as fact or opinion</td>
<td>4</td>
<td>High school textbook, college course notes</td>
<td>College library, course notes, high school teacher, college professor</td>
<td>8/1/15</td>
<td>8/1/15</td>
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<tr>
<td>Contextual meaning</td>
<td>Identify the meanings of words as they are used in the context of a reading selection</td>
<td>2</td>
<td>High school textbook, college course notes</td>
<td>College library, course notes, high school teacher, college professor</td>
<td>8/1/15</td>
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<tr>
<td>Figurative Language</td>
<td>Understand figurative language and nuances in word meanings</td>
<td>2</td>
<td>High school textbook, college course notes</td>
<td>College library, course notes, high school teacher, college professor</td>
<td>8/8/15</td>
<td>8/8/15</td>
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<tr>
<td>Vocabulary range</td>
<td>Understand a range of words and phrases sufficient for reading at the college and career readiness level</td>
<td>2</td>
<td>High school textbook, college course notes</td>
<td>College library, course notes, high school teacher, college professor</td>
<td>8/15/15</td>
<td>8/17/15</td>
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<tr>
<td>Integration of Knowledge and Ideas</td>
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<tr>
<td>Diverse media and formats</td>
<td>Analyze content presented in diverse media and formats, including visually and quantitatively, as well as in words</td>
<td>2</td>
<td>High school textbook, college course notes</td>
<td>College library, course notes, high school teacher, college professor</td>
<td>8/22/15</td>
<td>8/24/15</td>
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<tr>
<td>Evaluation of arguments</td>
<td>Identify the relationship among ideas presented in a reading selection</td>
<td>4</td>
<td>High school textbook, college course notes</td>
<td>College library, course notes, high school teacher, college professor</td>
<td>8/24/15</td>
<td>8/24/15</td>
</tr>
<tr>
<td>Evaluation of arguments</td>
<td>Determine whether evidence strengthens, weakens, or is relevant to the arguments in a reading selection</td>
<td>3</td>
<td>High school textbook, college course notes</td>
<td>College library, course notes, high school teacher, college professor</td>
<td>8/27/15</td>
<td>8/27/15</td>
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<tr>
<td>Evaluation of arguments</td>
<td>Determine the logical assumptions upon which an argument or conclusion is based</td>
<td>5</td>
<td>High school textbook, college course notes</td>
<td>College library, course notes, high school teacher, college professor</td>
<td>8/28/15</td>
<td>8/30/15</td>
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<tr>
<td>Evaluation of arguments</td>
<td>Draw conclusions from material presented in a reading selection</td>
<td>5</td>
<td>High school textbook, college course notes</td>
<td>College library, course notes, high school teacher, college professor</td>
<td>8/30/15</td>
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<tr>
<td>Comparison of texts</td>
<td>Recognize or predict ideas or situations that are extensions of or similar to what has been presented in a reading selection</td>
<td>4</td>
<td>High school textbook, college course notes</td>
<td>College library, course notes, high school teacher, college professor</td>
<td>9/3/15</td>
<td>9/4/15</td>
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<tr>
<td>Comparison of texts</td>
<td>Apply ideas presented in a reading selection to other situations</td>
<td>2</td>
<td>High school textbook, college course notes</td>
<td>College library, course notes, high school teacher, college professor</td>
<td>9/5/15</td>
<td>9/6/15</td>
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</tbody>
</table>
**My Study Plan**

Use this worksheet to:
1. **Define Content Areas**: List the most important content areas for your test as defined in chapter 1.
2. **Determine Strengths and Weaknesses**: Identify your strengths and weaknesses in each content area.
3. **Identify Resources**: Identify the books, courses, and other resources you plan to use for each content area.
4. **Study**: Create and commit to a schedule that provides for regular study periods.

<table>
<thead>
<tr>
<th>Praxis Test Name (Test Code):</th>
<th>________________________________________________________________________________</th>
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<tbody>
<tr>
<td>Test Date:</td>
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Step 5: Develop Your Study Plan

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6. Study Topics

Detailed study topics with questions for discussion

Using the Study Topics That Follow

The Elementary Education: Mathematics—CKT test is designed to measure the knowledge and skills necessary for a beginning elementary mathematics teacher.

This chapter is intended to help you organize your preparation for the test and to give you a clear indication of the depth and breadth of the knowledge required for success on the test.

Virtually all accredited programs address the topics covered by the test; however, you are not expected to be an expert on all aspects of the topics. You are likely to find that the topics that follow are covered by most introductory textbooks. Consult materials and resources, including lecture and laboratory notes, from all your coursework. For the mathematics CKT test, you will also find it helpful to identify student work samples and curriculum materials, which you could gather from your student teaching experience, from your own studies, or from education web sites. You should be able to match up specific topics and subtopics with what you have covered in your courses.

Try not to be overwhelmed by the volume and scope of content knowledge in this guide. Although a specific term may not seem familiar as you see it here, you might find you can understand it when applied to a real-life situation. Many of the items on the actual test will provide you with a context to apply to these topics or terms.

Study Questions

Interspersed throughout the content topics are study questions, intended to help test your knowledge of fundamental concepts and your ability to apply those concepts to situations in the classroom or the real world. Most of the areas require you to combine several pieces of knowledge to formulate an integrated understanding and response. If you spend time on these questions, you will gain increased understanding and facility with the subject matter covered on the test. You may want to discuss these questions and your answers with a teacher or mentor.

Note that this study companion does not provide answers for the study questions, but thinking about the answers to them will help improve your understanding of fundamental concepts and will probably help you answer a broad range of questions on the test.
Mathematics—CKT Study Topics

The mathematics component of the Elementary Education: Content Knowledge for Teaching test measures the content knowledge required to do the work of the elementary mathematics curriculum and the specialized content knowledge you must have to teach it. To prepare for the test, you may find it helpful first to review the content topics (e.g., I. Counting and Operations with Whole Numbers) and subtopics (e.g., A. Counting) below, making sure that you’re able to do the work commonly required of elementary students. Next, review each subtopic in the context of selected tasks of teaching, using the study questions to guide your review. The list of study questions is intended to help you tap into some of the specialized content knowledge you need to carry out the targeted tasks of teaching, but the list is not exhaustive. For a more thorough review of each subtopic, choose additional tasks of teaching from the complete list below and ask yourself, “What do I need to know about this content to be able to engage in this task of teaching?”

Tasks of Teaching Mathematics

This list includes tasks that are essential for effective teaching of elementary mathematics.

Explanations, Conjectures, and Definitions

1. Giving mathematically valid explanations for a process, conjecture, or relationship
2. Evaluating mathematical explanations for their validity, generalizability, explanatory power, and/or completeness
3. Determining the changes that would improve the validity, generalizability, completeness, and/or precision of a mathematical explanation
4. Evaluating a student conjecture for its validity and/or generalizability on a given domain
5. Evaluating mathematical definitions or other mathematical language for precision, validity, generalizability, usefulness in a particular context, and/or support for an instructional goal

Problems, Examples, and Structure

6. Evaluating mathematical problems for how well they elicit a particular idea, support the use of a particular solution strategy or practice, fit a particular mathematical structure, address the same concept as another problem, or assess a particular student conception or error
7. Writing mathematical problems that fit a particular solution strategy or mathematical structure
8. Evaluating examples for how well they introduce a concept; illustrate an idea or relationship; illustrate the appropriateness of a strategy, procedure, or practice; or address particular student questions, misconceptions, or partial conceptions
9. Generating or identifying nonexamples or counterexamples to highlight a mathematical distinction or to demonstrate why a student conjecture is incorrect or partially incorrect
10. Choosing which mathematical topics are most closely related to a particular instructional goal

Representations and Manipulatives

11. Selecting, creating, or evaluating representations or manipulatives for a mathematical purpose or to show a particular mathematical idea
12. Evaluating how representations or manipulatives have been used to show particular mathematical ideas, relationships between ideas, mathematical processes, or strategies in a text, talk, or written work

Student Strategies and Errors

13. Determining whether student work demonstrates the use of a particular mathematical idea or strategy
14. Determining whether a strategy is mathematically valid or generalizable
15. Interpreting a student’s mathematical error, including anticipating how it would replicate across similar problems, and choosing other work samples that demonstrate the same error
16. Identifying tasks or situations in which student work or talk that seems mathematically valid might mask incorrect thinking
Content Topics

I. Counting and Operations with Whole Numbers

This list details the mathematics topics critical for elementary students to master.

A. Counting
1. Counts and skip counts whole numbers between 0 and 1,000
2. Counts on, starting with any whole number
3. Connects counting to cardinality
4. Demonstrates understanding of one-to-one correspondence between numbers and objects being counted
5. Subitizes (recognizes small quantities by sight)
6. Identifies relationships between counting and the concept of larger and smaller numbers (i.e., that sets with higher counts are larger than sets with smaller counts)

<table>
<thead>
<tr>
<th>Study Question</th>
<th>Task Targeted by Study Question</th>
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<tbody>
<tr>
<td>What are some counting tasks that can be used to assess students’ understanding of the following key ideas in counting: one-to-one correspondence, counting out a particular quantity from a larger quantity, cardinality, conservation of cardinality, and ordinality?</td>
<td>6. Evaluating mathematical problems for how well they elicit a particular idea, support the use of a particular solution strategy or practice, fit a particular mathematical structure, address the same concept as another problem, or assess a particular student conception or error</td>
</tr>
<tr>
<td>What are some ways students might demonstrate evidence of understanding (or of not understanding) any of the key ideas in counting listed in the preceding question?</td>
<td>13. Determining whether student work demonstrates the use of a particular mathematical idea or strategy</td>
</tr>
</tbody>
</table>

B. Operations with Whole Numbers
1. Demonstrates understanding of representations of addition, subtraction, multiplication, and division (including objects such as manipulatives, drawings, and diagrams) and relates these representations of operations to expressions and equations
2. Solves mathematical and real-world problems involving the four operations, including solving problems by using properties of operations

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<thead>
<tr>
<th>Study Question</th>
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<tbody>
<tr>
<td>What are some examples of word problems that can be answered using addition or subtraction and that have a join structure, a separate structure, a part-part-whole structure, or a comparison structure? What are some examples of word problems that can be answered using addition or subtraction and in which the result is unknown, the initial amount is unknown, or the amount of change is unknown?</td>
<td>6. Evaluating mathematical problems for how well they elicit a particular idea, support the use of a particular solution strategy or practice, fit a particular mathematical structure, address the same concept as another problem, or assess a particular student conception or error</td>
</tr>
<tr>
<td>Write a word problem that uses a specified model of division (i.e., measurement or partitive) and a specified interpretation of the remainder (e.g., discard the remainder, remainder forces the answer to the next-highest whole number).</td>
<td>7. Writing mathematical problems that fit a particular solution strategy or mathematical structure</td>
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</table>
### Study Question

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<th>Study Question</th>
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<tr>
<td>Review the take-away and comparison interpretations of subtraction and various strategies for adding and subtracting (e.g., compensation, shifting the problem, etc.). How could you use moves on a number line to represent solutions to addition and subtraction problems using combinations of these interpretations and strategies? How could you use moves on a Rekenrek to represent the solution? (Note: Other manipulatives to be familiar with include Cuisenaire® rods, interlocking cubes, and bundling sticks.)</td>
<td>12. Evaluating how representations or manipulatives have been used to show particular mathematical ideas, relationships between ideas, mathematical processes, or strategies in a text, talk, or written work</td>
</tr>
<tr>
<td>Review some common strategies for multiplication, including finding partial products. How would you use various area models to represent these strategies?</td>
<td>12. Evaluating how representations or manipulatives have been used to show particular mathematical ideas, relationships between ideas, mathematical processes, or strategies in a text, talk, or written work</td>
</tr>
<tr>
<td>Think of a two-digit multiplication problem. What are strategies to solve the problem that use the commutative property, the associative property, the distributive property, or the place value of the numbers in the problem?</td>
<td>13. Determining whether student work demonstrates the use of a particular mathematical idea or strategy</td>
</tr>
<tr>
<td>Look at some different strategies students have used to multiply whole numbers. Which strategies work no matter what whole numbers are being multiplied? Which strategies only work for some whole numbers, and what whole numbers do these strategies work for? Do the same for addition, subtraction, and division strategies.</td>
<td>14. Determining whether a strategy is mathematically valid or generalizable</td>
</tr>
<tr>
<td>Think of errors students might make when adding, subtracting, multiplying, or dividing whole numbers. How would you describe these errors? Now focus on a particular error. How is a student who makes this error likely to answer another question asking him to perform the same operation on other whole numbers?</td>
<td>15. Interpreting a student’s mathematical error, including anticipating how it would replicate across similar problems, and choosing other work samples that demonstrate the same error</td>
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II. Place Value and Decimals

A. Place Value and Decimals

1. Demonstrates a conceptual understanding of the value of the digits in a number
2. Compares multidigit and decimal numbers
3. Rounds multidigit and decimal numbers
4. Composes and decomposes multidigit numbers into groupings and understands why grouping and ungrouping are helpful in performing operations on multidigit and decimal numbers
5. Uses drawings and objects such as manipulatives to represent place value, relating these drawings and objects to numerical equations and written descriptions

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<tr>
<td>What are some different strategies students might use to compare multidigit whole numbers or decimal numbers? Which strategies are mathematically valid?</td>
<td>2. Evaluating mathematical explanations for their validity, generalizability, explanatory power, and/or completeness</td>
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<tr>
<td>What are different ways that the number 3.4 could be represented using base ten blocks?</td>
<td>11. Selecting, creating, or evaluating representations or manipulatives for a mathematical purpose or to show a particular mathematical idea</td>
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<tr>
<td>Think of errors that students might make when rounding, comparing, adding, or subtracting decimals. What are some examples of problems in which the error would be evident, and what are some examples of problems in which the error would not be evident?</td>
<td>16. Identifying tasks or situations in which student work or talk that seems mathematically valid might mask incorrect thinking</td>
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III. Fractions, Operations with Fractions, and Ratios

A. Fractions, Operations with Fractions, and Ratios

1. Demonstrates understanding of fractions as part-whole relationships, as multiples of unit fractions, as numbers, and as ratios, moving back and forth flexibly among these conceptualizations
2. Demonstrates understanding of characteristics of fractions that are less than one, equal to one, and greater than one
3. Demonstrates understanding of equipartitioning and that it is a building block for understanding fractions as part-whole relationships
4. Demonstrates understanding of fraction equivalence
5. Uses a variety of strategies for comparing fractions
6. Performs operations such as addition, subtraction, multiplication, and division with fractions as well as with fractions and whole numbers, understanding and using different strategies for these operations and building intuition about how the operations work (e.g., recognizing that multiplying a whole number by a fraction that is less than one makes the product smaller)
7. Demonstrates understanding of applications of operations on fractions (e.g., scaling)

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<tr>
<td>Choose some common algorithms for working with fractions (e.g., “butterfly/cross multiply” method to compare fractions, invert and multiply to divide fractions). Then explain why each algorithm is mathematically valid.</td>
<td>1. Giving mathematically valid explanations for a process, conjecture, or relationship</td>
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<tr>
<td>Choose two fractions to multiply. Then explain how you could use an area model to represent the product of the two fractions.</td>
<td>1. Giving mathematically valid explanations for a process, conjecture, or relationship</td>
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<tr>
<td>Consider some key concepts of fractions (e.g., fractions are divided into equal parts, how to find equivalent fractions). What wording would need to be included in an explanation to correctly address the concept?</td>
<td>3. Determining the changes that would improve the validity, generalizability, completeness, and/or precision of a mathematical explanation</td>
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<td>What are some observations students might make about patterns they see when comparing, multiplying, or dividing fractions (e.g., the fraction with the bigger numerator is always the bigger fraction; when you divide a number by a fraction, the answer is always bigger than the original number)? For what types of fractions will the pattern hold?</td>
<td>4. Evaluating a student conjecture for its validity and/or generalizability on a given domain</td>
</tr>
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<td>What are different ways to name the fraction ( \frac{7}{10} ) (e.g., seven divided by ten, seven to ten)? What mathematical meaning is emphasized by each way to name the fraction?</td>
<td>5. Evaluating mathematical definitions or other mathematical language for precision, validity, generalizability, usefulness in a particular context, and/or support for an instructional goal</td>
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<td>How can benchmark numbers, such as, ( \frac{1}{2} ), be used when comparing fractions or performing operations with fractions?</td>
<td>8. Evaluating examples for how well they introduce a concept; illustrate an idea or relationship; illustrate the appropriateness of a strategy, procedure, or practice; or address particular student questions, misconceptions, or partial conceptions</td>
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<td>Study Question</td>
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<tr>
<td>Look up some word problems involving addition, subtraction, or multiplication of fractions. How could each problem be represented using an area model, a number line, or a tape diagram?</td>
<td>11. Selecting, creating, or evaluating representations or manipulatives for a mathematical purpose or to show a particular mathematical idea</td>
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<tr>
<td>Look at some samples of student work showing how to compare or add fractions. What strategies or abilities are demonstrated in the student work? What lack of understanding, if any, is demonstrated in the student work?</td>
<td>13. Determining whether student work demonstrates the use of a particular mathematical idea or strategy</td>
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<tr>
<td>Look at different strategies students have used to compare, multiply, or divide fractions. Determine whether the strategy works no matter what fractions are in the problem, and if not, what the limitations are.</td>
<td>14. Determining whether a strategy is mathematically valid or generalizable</td>
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<tr>
<td>Think of different incorrect answers students give when using an area model to represent a fraction or to compare fractions. What misconceptions might underlie those incorrect answers?</td>
<td>15. Interpreting a student’s mathematical error, including anticipating how it would replicate across similar problems, and choosing other work samples that demonstrate the same error</td>
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<tr>
<td>Think of a fraction addition problem. What are some errors students might make when solving the problem? Now focus on a particular error. How is a student who makes this error likely to solve a different problem asking him to add fractions? Do the same for subtraction, multiplication, and division problems with fractions.</td>
<td>15. Interpreting a student’s mathematical error, including anticipating how it would replicate across similar problems, and choosing other work samples that demonstrate the same error</td>
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IV. Early Equations and Expressions, Measurement, and Geometry

A. Early Equations and Expressions

1. Demonstrates understanding of what it means for algebraic terms, expressions, and equations to be considered equivalent, how the equal sign is used to represent relational equivalence, and that equations maintain their equivalence status under certain algebraic manipulations.

2. Determines whether equations are true, identifies the missing values that would make them true, solves equations using the four operations, and solves relational statements by substitution.

3. Follows the standard order of operations (including the use of parentheses and the distributive property of multiplication over addition).

4. Demonstrates awareness of different interpretations of the word “variable,” including the ideas of quantities that are unknown (which underlies understanding how to solve equations) and quantities that vary (which can be connected to patterns and will support later understanding of functional relationships).

5. Uses the less-than and greater-than relational symbols (<, >) to compare quantities.

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<tr>
<th>Study Question</th>
<th>Task Targeted by Study Question</th>
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<tbody>
<tr>
<td>Look at some samples of student explanations of properties of operations (e.g., commutative property, associative property, distributive property) or properties of numbers (e.g., odd, even, divisible by 5). For each explanation, does the student merely assume that the property is true without showing why, only give examples to show that the property is true, or actually show that the property is true in general?</td>
<td>2. Evaluating mathematical explanations for their validity, generalizability, explanatory power, and/or completeness</td>
</tr>
<tr>
<td>Think of different ways that the number of squares in the outside border of an ( n \times n ) square can be found without counting. Can you write an expression in terms of ( n ) to represent each method?</td>
<td>11. Selecting, creating, or evaluating representations or manipulatives for a mathematical purpose or to show a particular mathematical idea</td>
</tr>
<tr>
<td>Think of a two-step equation. What are some errors students might make when solving the equation? Now focus on a particular error. How is a student who makes this error likely to solve a different two-step equation? Do the same for one-step equations.</td>
<td>15. Interpreting a student’s mathematical error, including anticipating how it would replicate across similar problems, and choosing other work samples that demonstrate the same error</td>
</tr>
</tbody>
</table>
B. Measurement

1. Describes measurable attributes of objects
2. Compares two objects with a common measurable attribute
3. Chooses appropriate measurement tools and uses the tools to take measurements
4. Calculates and estimates perimeter, area, volume, and measurements of angles in mathematical and real-world problems
5. Converts between measurement units

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<td>What are some activities that can be used when introducing students to measurement? What measurement concepts are related to these activities?</td>
<td>6. Evaluating mathematical problems for how well they elicit a particular idea, support the use of a particular solution strategy or practice, fit a particular mathematical structure, address the same concept as another problem, or assess a particular student conception or error</td>
</tr>
<tr>
<td>Identify the examples that illustrate a measurement concept (e.g., direct measurement versus indirect measurement, standard units versus informal units).</td>
<td>8. Evaluating examples for how well they introduce a concept; illustrate an idea or relationship; illustrate the appropriateness of a strategy, procedure, or practice; or address particular student questions, misconceptions, or partial conceptions</td>
</tr>
<tr>
<td>Look at some samples of student work on or student responses to measurement problems (e.g., measuring length with a ruler; finding area, perimeter, or volume; converting between measurement units). What does each sample tell you about the student's understanding of the concepts assessed in the problem?</td>
<td>15. Interpreting a student's mathematical error, including anticipating how it would replicate across similar problems, and choosing other work samples that demonstrate the same error</td>
</tr>
</tbody>
</table>

C. Geometry

1. Demonstrates understanding of shapes and their attributes
2. Composes and decomposes shapes
3. Draws shapes based on specific attributes such as number of angles and number of equal faces
4. Demonstrates understanding of lines, line segments, rays, and angles in two-dimensional figures
5. Classifies two-dimensional figures based on properties

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<td>Look at different sets of quadrilaterals and identify a characteristic that the quadrilaterals have in common. What is the most comprehensive set of quadrilaterals that will still have the characteristic in common?</td>
<td>4. Evaluating a student conjecture for its validity and/or generalizability on a given domain</td>
</tr>
<tr>
<td>Think of different classifications of quadrilaterals (e.g., parallelogram, trapezoid, rectangle). What are some assumptions students might make about the characteristics of all the quadrilaterals in that classification? What, if any, are examples that would demonstrate that those assumptions are incorrect?</td>
<td>9. Generating or identifying nonexamples or counterexamples to highlight a mathematical distinction or to demonstrate why a student conjecture is incorrect or partially incorrect</td>
</tr>
</tbody>
</table>
7. Review Smart Tips for Success

**Follow test-taking tips developed by experts**

Learn from the experts. Take advantage of the following answers to questions you may have and practical tips to help you navigate the Praxis test and make the best use of your time.

**Should I guess?**

Yes. Your score is based on the number of questions you answer correctly, with no penalty or subtraction for an incorrect answer. When you don't know the answer to a question, try to eliminate any obviously wrong answers and then guess at the correct one. Try to pace yourself so that you have enough time to carefully consider every question.

**Can I answer the questions in any order?**

You can answer the questions in order or skip questions and come back to them later. If you skip a question, you can also mark it so that you can remember to return and answer it later. Remember that questions left unanswered are treated the same as questions answered incorrectly, so it is to your advantage to answer every question.

**Are there trick questions on the test?**

No. There are no hidden meanings or trick questions. All of the questions on the test ask about subject matter knowledge in a straightforward manner.

**Are there answer patterns on the test?**

No. You might have heard this myth: the answers on tests follow patterns. Another myth is that there will never be more than two questions in a row with the correct answer in the same position among the choices. Neither myth is true. Select the answer you think is correct based on your knowledge of the subject.

**Can I write on the scratch paper I am given?**

Yes. You can work out problems on the scratch paper, make notes to yourself, or write anything at all. Your scratch paper will be destroyed after you are finished with it, so use it in any way that is helpful to you. But make sure to select or enter your answers on the computer.

**Smart Tips for Taking the Test**

1. **Skip the questions you find extremely difficult.** Rather than trying to answer these on your first pass through the test, you may want to leave them blank and mark them so that you can return to them later. Pay attention to the time as you answer the rest of the questions on the test, and try to finish with 10 or 15 minutes remaining so that you can go back over the questions you left blank. Even if you don't know the answer the second time you read the questions, see if you can narrow down the possible answers, and then guess. Your score is based on the number of right answers, so it is to your advantage to answer every question.
2. **Keep track of the time.** The on-screen clock will tell you how much time you have left. You will probably have plenty of time to answer all of the questions, but if you find yourself becoming bogged down, you might decide to move on and come back to any unanswered questions later.

3. **Read all of the possible answers before selecting one.** For questions that require you to select more than one answer, or to make another kind of selection, consider the most likely answers given what the question is asking. Then reread the question to be sure the answer(s) you have given really answer the question. Remember, a question that contains a phrase such as “Which of the following does NOT …” is asking for the one answer that is NOT a correct statement or conclusion.

4. **Check your answers.** If you have extra time left over at the end of the test, look over each question and make sure that you have answered it as you intended. Many test takers make careless mistakes that they could have corrected if they had checked their answers.

5. **Don’t worry about your score when you are taking the test.** No one is expected to answer all of the questions correctly. Your score on this test is not analogous to your score on the GRE® or other tests. It doesn’t matter on the Praxis tests whether you score very high or barely pass. If you meet the minimum passing scores for your state and you meet the state’s other requirements for obtaining a teaching license, you will receive a license. In other words, what matters is meeting the minimum passing score. You can find passing scores for all states that use the Praxis tests at [www.ets.org/praxis/institutions/scores/passing/](http://www.ets.org/praxis/institutions/scores/passing/) or on the web site of the state for which you are seeking certification/licensure.

6. **Use your energy to take the test, not to get frustrated by it.** Getting frustrated only increases stress and decreases the likelihood that you will do your best. Highly qualified educators and test development professionals, all with backgrounds in teaching, worked diligently to make the test a fair and valid measure of your knowledge and skills. Your state painstakingly reviewed the test before adopting it as a licensure requirement. The best thing to do is concentrate on answering the questions.
8. Check on Testing Accommodations

See if you qualify for accommodations that may make it easier to take the Praxis test

What if English is not my primary language?

Praxis tests are given only in English. If your primary language is not English (PLNE), you may be eligible for extended testing time. For more details, visit www.ets.org/praxis/register/plne_accommodations/.

What if I have a disability or other health-related need?

The following accommodations are available for Praxis test takers who meet the Americans with Disabilities Act (ADA) Amendments Act disability requirements:

- Extended testing time
- Additional rest breaks
- Separate testing room
- Writer/recorder of answers
- Test reader
- Sign language interpreter for spoken directions only
- Perkins Brailler
- Braille slate and stylus
- Printed copy of spoken directions
- Oral interpreter
- Audio test
- Braille test
- Large print test book
- Large print answer sheet
- Listening section omitted

For more information on these accommodations, visit www.ets.org/praxis/register/disabilities.

Note: Test takers who have health-related needs requiring them to bring equipment, beverages, or snacks into the testing room or to take extra or extended breaks must request these accommodations by following the procedures described in the Bulletin Supplement for Test Takers with Disabilities or Health-Related Needs (PDF), which can be found at https://www.ets.org/s/praxis/pdf/bulletin_supplement_test_takers_with_disabilities_health_needs.pdf.

You can find additional information on available resources for test takers with disabilities or health-related needs at www.ets.org/disabilities.
9. Do Your Best on Test Day

*Get ready for test day so you will be calm and confident*

You followed your study plan. You prepared for the test. Now it’s time to prepare for test day.

Plan to end your review a day or two before the actual test date so you avoid cramming. Take a dry run to the test center so you’re sure of the route, traffic conditions, and parking. Most of all, you want to eliminate any unexpected factors that could distract you from your ultimate goal—passing the Praxis test!

On the day of the test, you should:

- be well rested
- wear comfortable clothes and dress in layers
- eat before you take the test
- bring an acceptable and valid photo identification with you
- be prepared to stand in line to check in or to wait while other test takers check in

You can’t control the testing situation, but you can control yourself. Stay calm. The supervisors are well trained and make every effort to provide uniform testing conditions, but don’t let it bother you if the test doesn’t start exactly on time. You will have the allotted amount of time once it does start.

You can think of preparing for this test as training for an athletic event. Once you’ve trained, prepared, and rested, give it everything you’ve got.

**What items am I restricted from bringing into the test center?**

You cannot bring into the test center personal items such as:

- handbags, knapsacks, or briefcases
- water bottles or canned or bottled beverages
- study materials, books, or notes
- pens, pencils, scrap paper, or calculators
- any electronic, photographic, recording, or listening devices

Personal items are not allowed in the testing room and will not be available to you during the test or during breaks. You may also be asked to empty your pockets. At some centers, you will be assigned a space to store your belongings, such as handbags and study materials. Some centers do not have secure storage space available, so please plan accordingly.

Test centers assume no responsibility for your personal items.
Step 9: Do Your Best on Test Day

If you have health-related needs requiring you to bring equipment, beverages or snacks into the testing room or to take extra or extended breaks, you need to request accommodations in advance. Procedures for requesting accommodations are described in the Bulletin Supplement for Test Takers with Disabilities or Health-related Needs (PDF).

Note: All cell phones, smart phones (e.g., Android® devices, iPhones®, etc.), and other electronic, photographic, recording, or listening devices are strictly prohibited from the test center. If you are seen with such a device, you will be dismissed from the test, your test scores will be canceled, and you will forfeit your test fees. If you are seen using such a device, the device will be confiscated and inspected. For more information on what you can bring to the test center, visit www.ets.org/praxis/test_day/bring.

Are You Ready?

Complete this checklist to determine whether you are ready to take your test.

☑ Do you know the testing requirements for the license or certification you are seeking in the state(s) where you plan to teach?

☑ Have you followed all of the test registration procedures?

☑ Do you know the topics that will be covered in each test you plan to take?

☑ Have you reviewed any textbooks, class notes, and course readings that relate to the topics covered?

☑ Do you know how long the test will take and the number of questions it contains?

☑ Have you considered how you will pace your work?

☑ Are you familiar with the types of questions for your test?

☑ Are you familiar with the recommended test-taking strategies?

☑ Have you practiced by working through the practice questions in this study companion or in a study guide or practice test?

☑ If constructed-response questions are part of your test, do you understand the scoring criteria for these questions?

☑ If you are repeating a Praxis test, have you analyzed your previous score report to determine areas where additional study and test preparation could be useful?

If you answered “yes” to the questions above, your preparation has paid off. Now take the Praxis test, do your best, pass it—and begin your teaching career!
Step 10: Understand Your Scores

10. Understand Your Scores

Understand how tests are scored and how to interpret your test scores

Of course, passing the Praxis test is important to you so you need to understand what your scores mean and what your state requirements are.

What are the score requirements for my state?
States, institutions, and associations that require the tests set their own passing scores. Visit www.ets.org/praxis/states for the most up-to-date information.

If I move to another state, will my new state accept my scores?
The Praxis tests are part of a national testing program, meaning that they are required in many states for licensure. The advantage of a national program is that if you move to another state that also requires Praxis tests, you can transfer your scores. Each state has specific test requirements and passing scores, which you can find at www.ets.org/praxis/states.

How do I know whether I passed the test?
Your score report will include information on passing scores for the states you identified as recipients of your test results. If you test in a state with automatic score reporting, you will also receive passing score information for that state.

A list of states and their passing scores for each test are available online at www.ets.org/praxis/states.

What your Praxis scores mean
You received your score report. Now what does it mean? It’s important to interpret your score report correctly and to know what to do if you have questions about your scores.

To access Understanding Your Praxis Scores, a document that provides additional information on how to read your score report, visit www.ets.org/praxis/scores/understand.

Put your scores in perspective
Your score report indicates:

- Your score and whether you passed
- The range of possible scores
- The raw points available in each content category
- The range of the middle 50 percent of scores on the test

If you have taken the same Praxis test or other Praxis tests over the last 10 years, your score report also lists the highest score you earned on each test taken.
Content category scores and score interpretation

Questions on the Praxis tests are categorized by content. To help you in future study or in preparing to retake the test, your score report shows how many raw points you earned in each content category. Compare your “raw points earned” with the maximum points you could have earned (“raw points available”). The greater the difference, the greater the opportunity to improve your score by further study.

Score scale changes

ETS updates Praxis tests on a regular basis to ensure they accurately measure the knowledge and skills that are required for licensure. When tests are updated, the meaning of the score scale may change, so requirements may vary between the new and previous versions. All scores for previous, discontinued tests are valid and reportable for 10 years, provided that your state or licensing agency still accepts them.

These resources may also help you interpret your scores:

- Understanding Your Praxis Scores (PDF), found at www.ets.org/praxis/scores/understand
- The Praxis Passing Scores (PDF), found at www.ets.org/praxis/scores/understand
- State requirements, found at www.ets.org/praxis/states
Appendix: Other Questions You May Have

Here is some supplemental information that can give you a better understanding of the Praxis tests.

What do the Praxis tests measure?

The Praxis tests measure the specific knowledge and skills that beginning teachers need. The tests do not measure an individual’s disposition toward teaching or potential for success, nor do they measure your actual teaching ability. The assessments are designed to be comprehensive and inclusive but are limited to what can be covered in a finite number of questions and question types. Teaching requires many complex skills that are typically measured in other ways, including classroom observation, video recordings, and portfolios.

Ranging from Agriculture to World Languages, there are more than 80 Praxis tests, which contain selected-response questions or constructed-response questions, or a combination of both.

Who takes the tests and why?

Some colleges and universities use the Praxis Core Academic Skills for Educators tests (Reading, Writing, and Mathematics) to evaluate individuals for entry into teacher education programs. The assessments are generally taken early in your college career. Many states also require Core Academic Skills test scores as part of their teacher licensing process.

Individuals entering the teaching profession take the Praxis content and pedagogy tests as part of the teacher licensing and certification process required by many states. In addition, some professional associations and organizations require the Praxis Subject Assessments for professional licensing.

Do all states require these tests?

The Praxis tests are currently required for teacher licensure in approximately 40 states and United States territories. These tests are also used by several professional licensing agencies and by several hundred colleges and universities. Teacher candidates can test in one state and submit their scores in any other state that requires Praxis testing for licensure. You can find details at www.ets.org/praxis/states.

What is licensure/certification?

Licensure in any area—medicine, law, architecture, accounting, cosmetology—is an assurance to the public that the person holding the license possesses sufficient knowledge and skills to perform important occupational activities safely and effectively. In the case of teacher licensing, a license tells the public that the individual has met predefined competency standards for beginning teaching practice.

Because a license makes such a serious claim about its holder, licensure tests are usually quite demanding. In some fields, licensure tests have more than one part and last for more than one day. Candidates for licensure in all fields plan intensive study as part of their professional preparation. Some join study groups, others study alone. But preparing to take a licensure test is, in all cases, a professional activity. Because a licensure test surveys a broad body of knowledge, preparing for a licensure test takes planning, discipline, and sustained effort.

Why does my state require the Praxis tests?

Your state chose the Praxis tests because they assess the breadth and depth of content—called the “domain”—that your state wants its teachers to possess before they begin to teach. The level of content knowledge, reflected in the passing score, is based on recommendations of panels of teachers and teacher educators in each subject area. The state licensing agency and, in some states, the state legislature ratify the passing scores that have been recommended by panels of teachers.
How were the tests developed?

ETS consulted with practicing teachers and teacher educators around the country during every step of the Praxis test development process. First, ETS asked them what knowledge and skills a beginning teacher needs to be effective. Their responses were then ranked in order of importance and reviewed by hundreds of teachers.

After the results were analyzed and consensus was reached, guidelines, or specifications, for the selected-response and constructed-response tests were developed by teachers and teacher educators. Following these guidelines, teachers and professional test developers created test questions that met content requirements and ETS Standards for Quality and Fairness.*

When your state adopted the research-based Praxis tests, local panels of teachers and teacher educators evaluated each question for its relevance to beginning teachers in your state. During this “validity study,” the panel also provided a passing-score recommendation based on how many of the test questions a beginning teacher in your state would be able to answer correctly. Your state’s licensing agency determined the final passing-score requirement.

ETS follows well-established industry procedures and standards designed to ensure that the tests measure what they are intended to measure. When you pass the Praxis tests your state requires, you are proving that you have the knowledge and skills you need to begin your teaching career.

How are the tests updated to ensure the content remains current?

Praxis tests are reviewed regularly. During the first phase of review, ETS conducts an analysis of relevant state and association standards and of the current test content. State licensure titles and the results of relevant job analyses are also considered. Revised test questions are then produced following the standard test development methodology. National advisory committees may also be convened to review and revise existing test specifications and to evaluate test forms for alignment with the specifications.

How long will it take to receive my scores?

Scores for tests that do not include constructed-response questions are available on screen immediately after the test. Scores for tests that contain constructed-response questions or essays aren’t available immediately after the test because of the scoring process involved. Official score reports are available to you and your designated score recipients approximately two to three weeks after the test date for tests delivered continuously, or two to three weeks after the testing window closes for other tests. See the test dates and deadlines calendar at www.ets.org/praxis/register/dates_centers for exact score reporting dates.

Can I access my scores on the web?

All test takers can access their test scores via My Praxis Account free of charge for one year from the posting date. This online access replaces the mailing of a paper score report.

The process is easy—simply log into My Praxis Account at www.ets.org/praxis and click on your score report. If you do not already have a Praxis account, you must create one to view your scores.

Note: You must create a Praxis account to access your scores, even if you registered by mail or phone.

Your teaching career is worth preparing for, so start today! Let the *Praxis* Study Companion guide you.

To search for the *Praxis* test prep resources that meet your specific needs, visit:

www.ets.org/praxis/testprep

To purchase official test prep made by the creators of the *Praxis* tests, visit the ETS Store:

www.ets.org/praxis/store