Summary, Overview, and Rationale for 2016 Revisions of NC Math I, II and III to NC Math 1, 2, and 3

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The High School Standards Revisions Writing Groups considered the work and recommendations of the *Academic Standards Review Commission*, teachers' comments from surveys and focus groups and the recommendations of the High School Data Review Committee in reviewing the standards related to number and quantity, algebra, functions, geometry and statistics & probability in the North Carolina Standard Course of Study. Overwhelmingly, all stakeholder groups who contributed comments on the standards requested clarification and a more coherent placement of standards; this to include course level clarification on

- the specific families of functions for focused study in the Algebra and Function strands;
- standards repeated across the three high school courses; and
- the wording and articulation of standards, to include the removal of non-applicable examples and notes.

The decision to move from a traditional sequence of Algebra 1, Geometry and Algebra 2 to the integrated approach allowed North Carolina to create opportunities for teachers and students to make mathematical connections throughout high school. There is now an opportunity for algebra skills to be used every year by continuing the integrated courses from K-8 through the first 3 maths in high school. This integrated organization of standards provides the same opportunities for advanced work in mathematics during high school, as does the Algebra 1, Geometry, Algebra 2 model.

Standards for Mathematical Practice

It should be noted that the Standards for Mathematical Practice (SMP) continue to be included as the foundation for reasoning mathematically in high school. Their inclusion in each course emphasizes the importance of providing opportunities throughout ALL content standards for students to analyze, argue, model, and problem solve in meaningful ways. Mathematics is a human activity of analyzing and solving problems and finding solutions; therefore, the SMP are critical for enabling students to learn mathematics in genuine ways. Important to the reading of the revision, is the understanding that the foundation of the Standards for Mathematical Practice are still intact. Therefore, the SMPs are listed in each course.

Modeling with mathematics also remains an integral part of all of the high school courses, in all of the content domains. While modeling with mathematics is the fourth SMP, we use the definition from The Consortium for Mathematics and its Applications (COMAP) and the Society for Industrial and Applied Mathematics (SIAM) to detail more specifically the process of mathematical modeling that the SMP bring to the content standards:

Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena.

(Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME), 2015)

Mathematical modeling is the way students connect the mathematical content they are learning to the real world in which they live. The vision for mathematics education in North Carolina is to ensure North Carolina students have mathematical understanding at or above the level of their national and international peers, ensuring that they are life, college and career ready.

Number and Quantity Conceptual Category

The number and quantity domain has been an area that has produced many questions and concerns. While these standards are essential to modeling and mathematics in general, the majority of the content in these standards is located throughout the Standards for Mathematical Practice. For this reason, N-Q.1, N-Q.2, and N-Q.3 will be completely integrated into the Standards for Mathematical Practice.

Focus and Rationale:

NC Math 1

In NC Math 1, students apply and extend their understanding of the number system from middle school to high school number and quantity. In 7th grade, students were formally introduced to the rational number system. In 8th grade, students were introduced to the real number system through the study of irrational numbers, approximating square and cube roots, and applying the properties of integer exponents to numerical expressions. This is significantly important to the systematic study of functions, which is a significant part of the HS standards. NC Math 1 students will apply their understanding and use of irrational numbers and square roots when solving simple quadratic equations by taking square roots. They will also apply their knowledge of the properties of integer exponents to algebraic expressions.

NC Math 2

In NC Math 2, students continue to build upon their knowledge of the properties of exponents by interpreting and using rational exponents. Students will use this knowledge to rewrite expressions with rational exponents or radicals into equivalent forms. Students will continue to investigate and formalize their understanding of the rational number system and irrational numbers. As students become fluent with quadratic equations in Math 2, they will become aware of the existence of complex numbers and use them as solutions to quadratic equations.

NC Math 3

In NC Math 3, students work with complex numbers and the Fundamental Theorem of Algebra as they study polynomial functions (quadratic and higher degree).

Algebra and Functions Conceptual Categories

The conceptual categories of Algebra and Functions are inter-related. Functions describe situations in which one quantity determines the other. The difference between the Function standards and the Algebra standards is that the Function standards focus more on the characteristics of functions, e.g., domain/range, function definition, etc. whereas the Algebra standards provide the computational tools and understandings that students need to explore specific instances of functions. As students progress through high school, the coursework with specific families of functions and algebraic manipulation evolve. For example the focus in NC Math 1 is on linear, quadratic, and exponential function families along with the algebraic concepts of solving equations, factoring quadratics, etc.

Algebra Conceptual Category

The Algebra Conceptual Category serves the purpose of connecting the generalizations of arithmetic operations with the function and geometric concepts across the standards. The learning in each course will align to the study of function types in each HS math course.

Focus and Rationale:

NC Math 1

The focus of algebra in NC Math 1 is on linear, exponential, and quadratic expressions/equations. NC Math 1 students will recognize the difference between the distinct expression types and make connections between expressions, equations, and functions. They build on the understanding of arithmetic operations. Solving equations and the basic introduction of linear equations was established in the 8th grade where students extensively studied linear and nonlinear functions. The goal of NC Math 1 is for students to develop a thorough understanding of linear functions as they begin studying exponential and quadratic equations/functions; viewing exponential and quadratic equations as extensions of nonlinear functions learned in the previous course. The focus in NC Math 1 is on these three equation types and the operations involved with them. Solving quadratic equations by factoring is now included in NC Math 1 and the appropriate related standards have been moved into the course. The quadratic formula is not an acceptable tool for factoring at the Math 1 level. This is to address a more coherent learning of quadratics in HS mathematics. It is expected that these equations come from a modeling context, when appropriate.

NC Math 2

The focus of algebra in NC Math 2 is on extending work with quadratic equations that were introduced in the previous HS course. The body of work for quadratics was previously split between the three HS math courses. The major work of quadratics has been moved to NC Math 2, including solving quadratics with complex solutions and completing the square to rewriting a quadratic in vertex form. Students will use rational exponents to rewrite radical expressions in an equivalent form. This concept has been moved from NC Math 1 to align with algebraic methods of solving radical equations. Inverse variation remains a focus of study in NC Math 2.

NC Math 3

The focus of algebra in NC Math 3 is to allow students to recognize the connection between arithmetic operations and equations. When entering Math 3, students should have mastery of linear, exponential, quadratic, and radical relationships, as well as inverse variation. Students will apply their knowledge of

these equations and functions to solving polynomials with the highest degree of 3 and solving absolute value equations and inequalities algebraically and graphically; to the relationship between exponential and logarithmic equations; and to rational equations limited to denominators with linear expressions. It is expected that these equations come from modeling contexts, when appropriate.

Functions Conceptual Category

Since elementary school, students have been working with patterns and focusing on the change from one term to the next in a sequence. In the middle grades, students learn to relate change across two patterns through ratio tables and to quantify this relationship with a rate. In grade 8, students are formally introduced to the concept of a function. They learn that a function describes a relationship between an input and output value, identify examples of function and non-functions, and develop an understanding of the family of linear functions. Students' work with function in high school should be seen as a next step in the development and refinement of their study of relationships between two varying quantities. The function families that are the focus of each course are listed in the diagram below:

NC Math 1	NC Math 2	NC Math 3
Linear	Quadratic	Exponential
Exponential	Square Root	Logarithm
Quadratic	Inverse Variation	Rationals w/ linear denominator
Functions	Functions	Polynomial w/ degree $\leq$ three
		Absolute Value and Piecewise

Note that the table is meant to show a progression so that students' understanding of each function family evolves over the three courses. In 8th grade, students are introduced to the notion of linear functions as they contrast with other functions in general. In NC Math 1, families of exponential and quadratic functions are added to the function types that students now explore. Students should become fluent in quadratic functions by Math 2, along with square root and inverse functions. Finally, in Math 3, exponential functions are revisited with logarithmic, rational, absolute value, piecewise, and polynomial functions added.

Focus and Rationale:

NC Math 1

In NC Math 1, students apply and extend their understandings of functions from grade 8 to the formal definition of a function and the use of function notation when expressing functions symbolically. They recognize that, in general, sequences defined recursively or explicitly are functions, and that arithmetic sequences are linear functions and that geometric sequences are exponential functions. They interpret and analyze different representations of a broad array of functions with a concentration on linear and exponential functions. Additionally, NC Math 1 students use recursive processes to build a variety of functions, construct and use explicit linear and exponential functions, and compare linear and exponential function models by interpreting the parameters of each model in terms of the context they represent.

NC Math 2

In NC Math 2, students apply and extend their understandings of functions from NC Math 1 by fluently operating with quadratic functions, building their repertoire of function families to include square root, and inverse functions.

NC Math 3

In NC Math 3, students use their learning about functions from previous courses to engage with a wider range of function families, including polynomial, rational, trigonometric, absolute value and piecewise-defined functions. Students find inverse functions and begin to explore the cosine and sine functions graphically. In various 4^{th} level math course options, students will have opportunities to reach fluency with the function families introduced in the course.

Geometry Conceptual Category

Included in the Geometry Conceptual Category is a 21st Century approach to the concepts of congruence and similarity. Learning geometry from a transformational point of view provides students with the 21st century tools and understanding to use their geometric knowledge in STEM-related fields from engineering to computer science to data transfer. This current positioning of geometry across the standards of NC Math 1, 2, and 3 allows us to help students build connections between their geometric understanding and mathematics as a whole.

Concepts of congruence and similarity are developed through the use of transformations (translations, rotations, reflections, and dilations) which gives students a visual and kinesthetic means of discussing and understanding geometric ideas. Given the importance of geometric ideas across the standards, The High School Data Review Committee & Writing Team recognized the need to articulate clearly where geometry topics are situated in the standards revision, why they are positioned in each course, and how they connect to the overall themes in each of the courses NC Math 1, NC Math 2, and NC Math 3.

NC Math 1	NC Math 2	NC Math 3
Focus on coordinate geometry • Distance on the coordinate plane • Midpoint of line segments • Slopes of parallel and perpendicular lines • Prove geometric theorems algebraically • Rational: Students make connections between algebra & geometry concepts via the coordinate plane	Focus on triangles • Congruence • Similarity moved from NC Math 3 to provide coherence • Right triangle trigonometry • Add special right triangles (assessed on the ACT)	Focus on circles • Move all content related to circles from NC Math 1 and NC Math 2 to NC Math 3 to provide coherence • Introduce the concept of radian • Introduce periodic functions but leave main work of trigonometric functions in 4th level math courses.
Additional Considerations		
Move the development of area and volume formulas to MS	Introduce proof • Students should use both informal and formal methods of reasoning to prove theorems related to lines, angles, and triangles	Formalize Proof • Focus on both paragraph and flow proofs
	Continue development of concepts around parallel lines and angles introduced in MS; important for proving theorems about triangles	Capstone learning with the connection of algebraic and geometric concepts through solving modeling and design problems

Constructions were removed as standards. They will be addressed in curricular resources as an instructional method since they are used to support student learning of concepts.

Developmentally, this is also appropriate—the work of Dina van Hiele-Geldof and her husband, Pierre van Hiele led to the development of *van Hiele Levels of Geometric Understanding.* These levels help us understand how students come to learn geometry. Students need incremental introduction to the topics of geometry with increasing sophistication that allows them to build on their concrete experiences with shapes and space and to identify relationships between properties of shapes. Students' ability to reason both informally and formally about the logical structure of geometry develops as they are more able to handle abstractions.

Focus and Rationale:

NC Math 1

The focus of geometry in NC Math 1 is ***coordinate geometry***. Students establish relationships and verify/prove them using coordinates and algebraic reasoning. The connections to the coordinate plane, distance, midpoint, and slope fit nicely with the emphasis on linear functions and algebraic reasoning in NC Math 1. The conceptual development of area and volume formulas will be moved to middle school when they are introduced and first used. Likewise, we will move the use of area and volume formulas to solve problems to Math 3 to support the goal of geometric modeling and design.

NC Math 2

The focus of geometry in NC Math 2 is on ***geometric relationships and properties of shape, focusing on lines, angles, and triangles***. Students continue to build on the middle school development of relationships about lines and angles to support proof of theorems about triangles. ***Triangles*** are the most fundamental two-dimensional polygon; an in-depth understanding of triangles will support the study of other polygons and circles in NC Math 3. ***Congruence*** and ***similarity*** are developed through a ***transformational approach***. Students are introduced to the ideas of ***geometric proof***, using both informal and formal methods of proof to develop and organize logical arguments. Similarity is utilized to develop ***right triangle trigonometry***, and special right triangles are introduced.

NC Math 3

The focus of geometry in NC Math 3 is on ***circles***. Students continue their study of relationships in polygons by ***conjecturing and proving relationships about quadrilaterals***. The goal in proof is to further develop the ability to construct logical arguments and to develop both ***flow*** and ***paragraph proofs***. Students develop ideas and properties about circles, including the idea of ***radians***. Periodic functions are introduced but the main work with trigonometric functions is housed in the fourth math courses. There is opportunity for ***capstone learning***— the connection of students' ideas of algebra, geometry, and functions—through solving ***modeling and design problems***.

General notes:

The writing team developed a ***list of the theorems*** that are to be proven in each course. This list is not exhaustive, but the standard. Proving theorems should include the relationships on the list, but are not limited to those alone. The expectation is that proofs will emphasize paragraph and flow proofs. The traditional 2-column proof is not the expectation. The construction of logical arguments and the ability to explain their reasoning is what will be expected from students.

Statistics and Probability Conceptual Category

In an increasingly data-driven world, statistical literacy is becoming an essential competency, not only for researchers conducting formal statistical analyses, but for informed citizens making everyday decisions based on data. Whether following media coverage of current events, making financial decisions, or assessing health risks, the ability to process statistical information is critical for navigating modern society.

(Franklin et al, *Statistical Education of Teachers*, 2014)

As informed citizens of an "increasingly data-driven world" students will need to be able to calculate useful statistics and probabilities, as well as understand what those calculations mean about the data within the given context. They will be asked to create and utilize representations and models of data. Further, students will need to be critical of the process of statistical investigation which influences the validity of sample statistics as representative of a population.

Focus and Rationale:

NC Math 1

The focus of *Statistics and Probability* in NC Math 1 includes analysis of univariate and bivariate data. With univariate data students will calculate and use statistics measuring center and spread that describe characteristics of a data set and that distinguish one data set from another. With bivariate data students will use scatterplots to study associations between variables. Regression is limited to linear and exponential functions.

Students will have created graphical representations of univariate data in 8th grade so that in NC Math 1 they will be ready to utilize technology to create such representations efficiently. Instructional focus should be placed on interpreting the characteristics of the data seen within the representation or using the representations to compare and contrast multiple data sets.

The course treatment of bivariate data includes the creation of scatterplots and the creation of functions to fit the plots. Students should be asked to consider only linear and exponential functions as potential tools for modeling the data.

Before fitting a linear function to a data set, students will describe the possible association of the data values (strongly or weakly associated, positively or negatively associated) that is seen from the scatterplot. Then when determining a linear function that fits a scatterplot, students will consider residual values as an indication of the goodness of fit of the linear function.

The American Statistical Association's *GAISE Report* (2007) places the understanding of Pearson's correlation coefficient as a tool for students who are working within the most sophisticated level of school statistics (there are 3 levels in the framework provided in the report). Analyzing residuals is a first step toward understanding Pearson's correlation coefficient as a tool for determining goodness of fit, thus supporting students' development toward that highest level within the framework.

An important part of the analysis of bivariate data and the potential association of the variables is to distinguish any observed association from causation. This requires students to consider data in its context to determine if causation is reasonable or not.

A final note about calculation and technology-
Students will benefit from calculating some number of standard deviations and residuals by hand, but technology should also be used for more efficient calculations in order to move the instructional focus to an analysis of the results of those calculations. Students should use technology to find equations of least squares regression lines or least squares exponential curves.

NC Math 2
The focus of *Statistics and Probability* in NC Math 2 is on probability. Students will compare and contrast experimental and theoretical probabilities, and within and without contexts investigate the concepts of independent events and conditional probabilities. Students will develop their understanding of the concepts of independent events and conditional probabilities, as well as apply rules of calculating probabilities that include conditional probabilities and independent events. Students will also use two-way tables of categorical data to calculate probabilities.

The *Statistics and Probability* standards for NC Math 2 include the majority of the content of probability for secondary mathematics. It is important for teachers to note that students experienced probability content in Grade 7 math, where the experience included work in both theoretical and experimental probabilities. It is in NC Math 2 that students should use the language of probability.

Several rules of probabilities are to be utilized by students and can be verified with examples. These calculations should be connected back to the context of the problem.

NC Math 3
The focus of *Statistics and Probability* in NC Math 3 is on the use of sample data to represent a population. Important understandings include the necessity of randomization in sampling to ensure an unbiased representation of a population. Students will engage in simulation to create a random sample of a population, calculating sample statistics representative of population statistics. Students will examine and evaluate the design and results of population studies that utilize samples.

Students need to be informed about the appropriate design and implementation of studies that utilize sample statistics to represent a population. This course will introduce students to the process of inference: that is, drawing conclusions about a population based on information from a sample. Students will use simulation to see that the variability in samples can be used to predict a margin of error. This work will be based on simulation only, not on any computational formulas. By the end of the course students should understand how statistics may be used to analyze data and make decisions.

References

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