Summary, Overview and Rationale of 2016 Revisions for NC Math 1, 2, and 3

Draft: April 18, 2016

The High School Standards Revisions Writing Groups considered teachers' comments from surveys and focus groups and the recommendations of the High School Data Review Committee in reviewing the standards related to number and quantity, algebra, functions, geometry and statistics & probability in the North Carolina Standard Course of Study. Overwhelmingly, all stakeholder groups who contributed comments on the standards requested clarification and a more coherent placement of standards; this to include course level clarification on a) the specific families of functions for focused study in the Algebra and Function strands, b) standards repeated across the three high school courses, and c) the wording and articulation of standards, to include the removal of non-applicable examples and notes.

Many people have not understood the decision to move from a "traditional" sequence of *Algebra 1*, *Geometry*, and *Algebra 2* to the more integrated approach of *NC Math 1, 2 and 3*. There are many reasons for the transition and shift. The courses of algebra and geometry that we think of as "traditional" are actually creations of the US educational system and are traditional only to our country and culture. Throughout history and across the world, students successfully study mathematics (many cultures refer to the discipline as *maths,* in the plural) that works to build connections across the topics of geometry, algebra, modeling, and statistics.

Number and Quantity Strand

Rationale:
The quantity domain has been an area that has produced many questions and concerns. While these standards are essential to modeling and mathematics in general, the majority of the content of these standards are located throughout the Standards for Mathematical Practice. For this reason, N-Q.1, N-Q.2, and N-Q.3 will be completely integrated into the Standards of Mathematical Practice and in the standards that support modeling.

Focus:
In NC Math 1, students apply and extend their understanding of the number system from middle school to high school number and quantity. In 7th grade, students were formally introduced to the rational number system. In 8th grade, students were introduced to the real number system through the study of irrational numbers, approximating square and cube roots, and applying the properties of integer exponents to numerical expressions. This is significantly important to the systematic study of functions, which is a significant part of the HS standards. NC Math 1 students will apply their understanding and use of irrational numbers and square roots when solving simple quadratic equations by taking square roots. They will also apply their knowledge of the properties of integer exponents to algebraic expressions. New to NC Math 1, students will be expected to use and interpret matrices by performing operations through addition, subtraction and scalar multiplication.

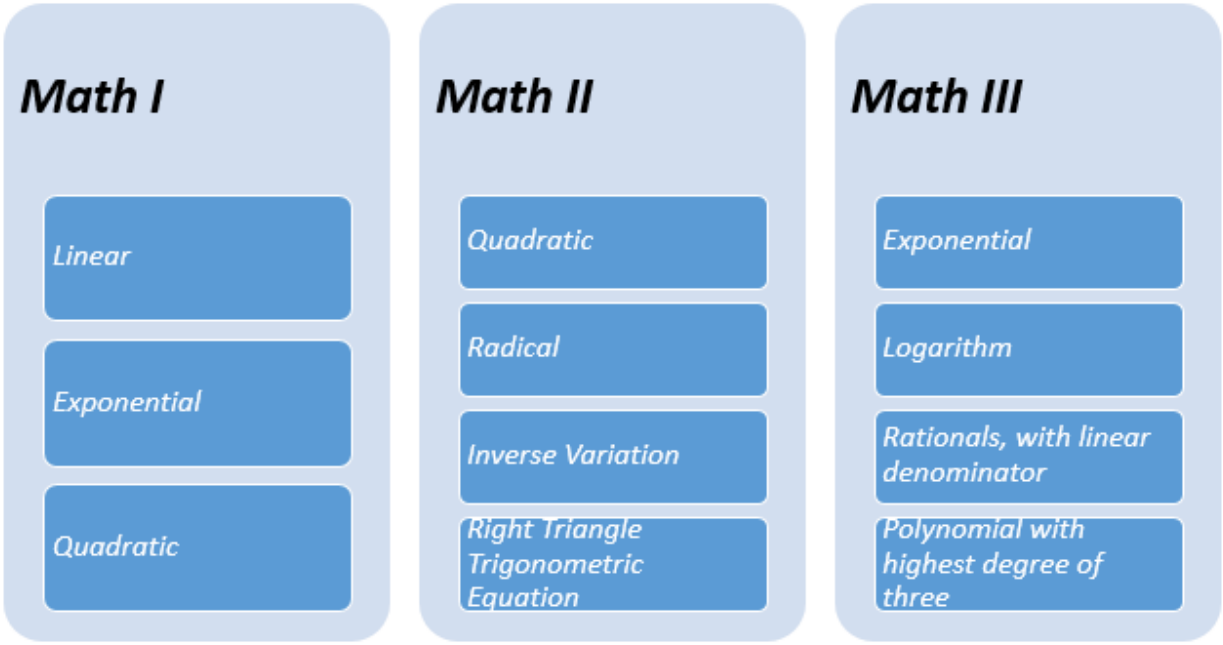
In NC Math 2, students continue to build upon their knowledge of the properties of exponents by interpreting and using rational exponents. Students will use this knowledge to rewrite expressions with rational exponents or radicals into equivalent forms. Students will continue to investigate and formalize their understanding of the rational number system and irrational numbers. As quadratic equations are mastered in this course, students will be introduced to complex numbers and the Fundamental Theorem of Algebra.

In NC Math 3, students continue their work with complex numbers and the Fundamental Theorem of Algebra as they study polynomial functions.

Algebra Strand

Rationale:

The Algebra Strand serves the purpose of connecting the learning of arithmetic operations with the concepts of functions and geometry. The learning in each course will align to the study of function types in each HS math course.



Focus:

The focus of algebra in NC Math 1 is on linear, exponential, and quadratic expressions/equations. NC Math 1 students recognize the difference between the distinct expression types and make connections between expressions, equations, and functions. They build on the understanding of arithmetic operations, solving equations and the basic introduction of linear equations established in the 8th grade where students extensively studied linear and nonlinear functions. The goal of NC Math 1 is for students to develop a thorough understanding of linear functions as they begin studying exponential and quadratic equations/functions; viewing exponential and quadratic equations as extensions of nonlinear functions learned in the previous course. The focus in NC Math 1 is on these three equation types and the operations involved with them. A coherence issue that was found in previous standards was that students were asked to algebraically reveal the zeroes by factoring a quadratic expression, however they were not supposed to solve quadratic equations by factoring. Therefore, solving quadratic equations by factoring is now included in NC Math 1 and the appropriate related standards have been moved into the course. It is expected that these equations come from a modeling contexts, when appropriate.

The focus of algebra in NC Math 2 is more extensive work with quadratic equations introduced in the previous HS course. The body of work for quadratics was previously split between the three HS math

courses. The major work of quadratics has been moved to NC Math 2, including solving quadratics with complex solutions and completing the square to rewrite a quadratic in vertex form. Trigonometric equations are also introduced in NC Math 2. This builds from the concepts of ratio and proportional reasoning developed in the middle school math courses. The trigonometric ratios of sine, cosine, and tangent should be viewed as unit rates of trigonometric equations. This should also connect to the concepts in the Geometry strand with similar triangles. Students will use rational exponents to rewrite radical equations in an equivalent form. This concept has been moved from NC Math 1 to align with algebraic methods of solving radical equations. Inverse variation remains a focus of study in NC Math 2. Inverse variation is defined by a constant of proportionality divided by a monomial. Absolute value equations are introduced in NC Math 2; students are only expected to solve these equations graphically. Solving absolute value equations algebraically will remain in the 4th level math courses. It is expected that these equations come from modeling contexts, when appropriate.

The focus of algebra in NC Math 3 is to allow students to recognize the connection between arithmetic operations and equations. At this point, students should have mastery of linear, exponential, quadratic, radical, inverse variation, and right triangle trigonometric equations. Students will extend their knowledge of these equation and functions to solving polynomials with the highest degree of 3, the relationship between exponential and logarithmic equations, and rational equations limited to a common denominator of a linear expression. It is expected that these equations come from modeling contexts, when appropriate.

Functions Strand

Rationale:
Since elementary school, students have been working with patterns and focusing on the change from one term to the next in a sequence. In middle grades, students learn to relate change across two patterns through ratio tables and to quantify this relationship with a rate. In grade 8, students are formally introduced to the concept of a function. They learn that a function describes a relationship between an input and output value, identify examples of function and non-functions, and develop an understanding of the family of linear functions. Students' work with function in high school should be seen as a next step in the development and refinement of their study of relationships between two varying quantities.

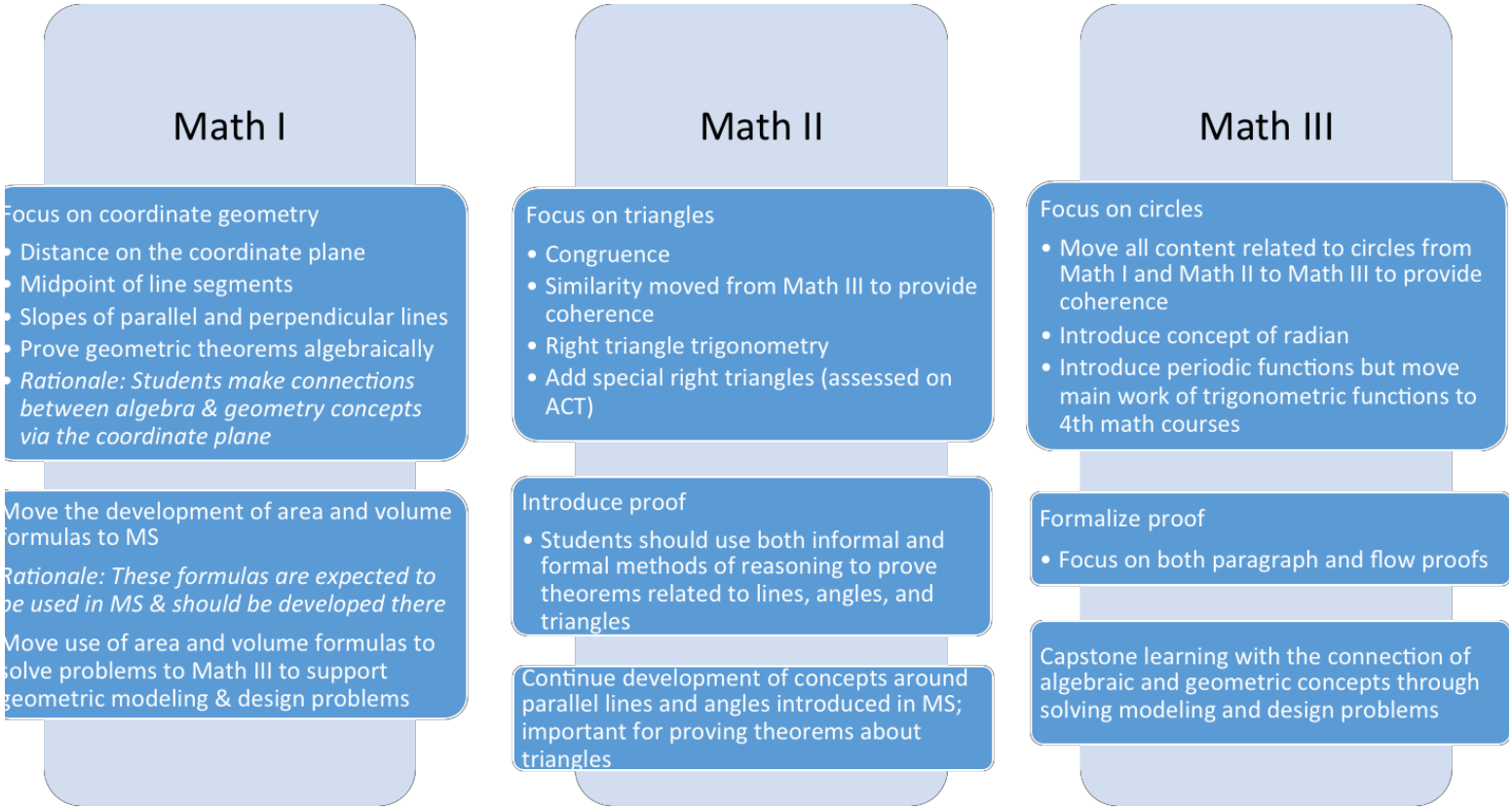
Focus:
In NC Math 1, students apply and extend their understandings of functions from grade 8 to the formal definition of a function and the use of function notation when expressing functions symbolically. They recognize that, in general, sequences defined recursively or explicitly are functions, and that arithmetic sequences are linear functions and geometric sequences are exponential functions. They interpret and analyze different representations of a broad array of functions with a concentration on linear and exponential functions. Additionally, NC Math 1 students use recursive processes to build a variety of functions, construct and use explicit linear and exponential functions, and compare linear and exponential function models by interpreting the parameters of each model in terms of the context they represent.

In NC Math 2, students apply and extend their understandings of functions from NC Math 2 by fluently operating with quadratic functions, building their repertoire of function families to include absolute value, and square root functions, and recognizing geometric transformations as functions.

In NC Math 3, students use their learning about functions from previous courses to engage with a wider range of function families, including polynomial, rational, trigonometric, and piecewise-defined functions. Students find inverse functions and begin to explore the cosine and sine functions graphically. In various 4^{th} level math course options, students will have opportunities to reach fluency with the function families introduced in the course.

Geometry Strand

The Geometry strand takes the traditional Euclidean geometry course and moves it into the 21st Century. Concepts of congruence and similarity are developed through the use of transformations (translations, rotations, reflections, and dilations), which gives students a visual and kinesthetic means of grappling with geometric ideas. Given the importance of geometric ideas across the curriculum, The High School Data Review Committee recognized the need to articulate clearly where geometry topics are situated in the curriculum revision, why they are positioned in each course, and how they connect to the overall themes in each of the courses NC Math 1, NC Math 2, and NC Math 3.



Remove the majority of constructions and add to the unpacking as an instructional method where appropriate. Constructions help students learn concepts; do not need to be assessed in and of themselves.

Rationale:

Learning geometry from a transformational point of view provides students with the 21st century tools and understanding in order to use their geometric knowledge in STEM-related fields from engineering to computer science to data transfer. This current positioning of geometry across the curriculum of NC Math 1, 2, and 3 allows us to help students build connections between their geometric understanding and mathematics as a whole.

Developmentally, this is also appropriate—the work of Dina van Hiele-Geldorf and her husband, Pierre van Hiele led to the development of *van Hiele Levels of Geometric Understanding*. These levels help us understand how students come to learn geometry. Students need incremental introduction to the topics of geometry with increasing sophistication that allows them to build on their concrete experiences with shapes & space and to identify relationships between properties of shapes. Students' ability to reason both informally and formally about the logical structure of geometry develops as they are more able to handle abstractions.

Focus:

The focus of geometry in NC Math 1 is ***coordinate geometry***. Students establish relationships and verify/prove them using coordinates and algebraic reasoning. The connections to the coordinate plane, distance, midpoint, and slope fit nicely with the emphasis on linear functions and algebraic reasoning in NC Math 1. Our suggestion is to move the conceptual development of area and volume formulas to Middle School when they are introduced and first used. Likewise, we move the use of area and volume formulas to solve problems to Math 3 to support the goal of geometric modeling and design.

The focus of geometry in NC Math 2 is on ***geometric relationships and properties of shape, focusing on lines, angles, and triangles***. Students continue to build on the middle school development of relationships about lines and angles to support proof of theorems about triangles. ***Triangles*** are the most fundamental two-dimensional polygon; an in-depth understanding of triangles will support the study of other polygons and circles in NC fMath 3. ***Congruence*** and ***similarity*** are developed through a ***transformational approach***. Students are introduced to the ideas of ***geometric proof***, using both informal and formal methods of proof to develop and organize logical arguments. Similarity is utilized to develop ***right triangle trigonometry***, and special right triangles are introduced.

The focus of geometry in NC Math 3 is ***circle geometry***. Students continue their study of relationships in polygons by ***conjecturing and proving relationships about quadrilaterals***. The goal in proof is to develop further the ability to construct logical arguments and to develop both flow chart and ***paragraph proofs***. Students develop ideas and properties about circles, including the idea of ***radians***. Periodic functions are introduced but the main work with trigonometric is housed in the 4th math courses. The opportunity for ***capstone learning***—connecting students' ideas of algebra, geometry, and functions—is through solving ***modeling and design problems***.

General notes:

The writing team developed a ***list of the theorems*** that are to be proven in each course. This list is not exhaustive but the standard. Proving theorems should include the relationships on the list, but are not limited to those alone.

Across the state and country, the use of transformations to develop and understand congruence and similarity is relatively new. We need to provide professional development to build capacity for viewing these topics through a transformational lens. This includes:

- Isometries—transformations that preserve both distance and angle measure
- Proof of congruence criteria (SAS, AAS, SSS) through a transformational approach
- Uses of transformation to prove properties and to solve problems
- Representation of transformations with functions

Statistics and Probability Strand

In an increasingly data-driven world, statistical literacy is becoming an essential competency, not only for researchers conducting formal statistical analyses, but for informed citizens making everyday decisions based on data. Whether following media coverage of current events, making financial decisions, or assessing health risks, the ability to process statistical information is critical for navigating modern society.

(Franklin et al, *Statistical Education of Teachers*, 2014)

NC Math 1

Focus:
The focus of *Statistics and Probability* in NC Math 1 includes analysis of univariate and bivariate data. With univariate data students will calculate and use statistics measuring center and spread that describe characteristics of a data set and that distinguish one data set from another. With bivariate data students will use scatterplots to study associations between variables. Regression is limited to linear and exponential functions.

Rationale:
Students will have created graphical representations of univariate data in 8th grade, so that in NC Math 1 they will be ready to utilize technology to create such representations efficiently so that instructional focus can be placed on interpreting the characteristics of the data seen within the representation or using the representations to compare and contrast multiple data sets. Teachers may find that reviewing the mean absolute deviation that students learned in 6th grade a useful connection on which to build standard deviation in NC Math 1.

The course treatment of bivariate data includes the creation of scatterplots and the creation of functions to fit the plots. Students should be asked to consider only linear and exponential functions as potential tools for modeling the data.

Before fitting a linear function to a data set, students will describe the possible association of the data values (strongly or weakly associated, positively or negatively associated) that is seen from the scatterplot. Then when determining a linear function that fits a scatterplot, students will consider residual values as an indication of the goodness of fit of the linear function. If the graph of the residual values does not appear random, then the data points sit away from the fitted line in a predictable way and thus the data is not linearly associated.

The American Statistical Association's *GAISE Report* (2007) places the understanding of Pearson's correlation coefficient as a tool for students who are working within the most sophisticated level of school statistics (there are 3 levels in the framework provided in the report). Analyzing residuals is a first step toward understanding Pearson's correlation coefficient as a tool for determining goodness of fit, thus supporting students' development toward that highest level within the framework.

An important part of the analysis of bivariate data and the potential association of the variables is to distinguish any observed association from causation. This requires students to consider data in its context to determine if causation is reasonable or not.

A final note about calculation and technology-
Students will benefit from calculating some number of standard deviations and residuals by hand, but technology should also be used for more efficient calculations in order to move the instructional focus to an analysis of the results of those calculations. Students should use technology to find equations of least squares regression lines or least squares exponential curves.

NC Math 2

Focus:
The focus of *Statistics and Probability* in NC Math 2 is on probability. Students will compare and contrast experimental and theoretical probabilities, and within and without contexts investigate the concepts of independent events and conditional probabilities. Students will develop their understanding of the concepts of independent events and conditional probabilities, as well as apply rules of calculating probabilities that include conditional probabilities and independent events. Students will also use two-way tables of categorical data to calculate probabilities.

The *Statistics and Probability* standards for NC Math 2 include the majority of the content of probability for secondary mathematics. It is important for teachers to note that students may not have experienced probability content since their 7th grade math class, where the experience included work in both theoretical and experimental probabilities. It is in NC Math 2 that students should use the language of probability.

Students will use contexts and two-way tables to develop the understanding that the conditional probability of event A given event B is the likelihood that A will occur given that B has occurred. Student understanding of the concept of independent events will connect to their understanding of conditional probability; the conditional probability indicates independence. This connection provides a more intuitive understanding of the independence of two events than the fact that events are independent if the probability of both events occurring is the product of their probabilities. Thus, the relationship with $P(A \cap B) = P(A)P(B)$ is utilized as a rule of probability more so than a defining characteristic.

Several rules of probabilities are to be utilized by students and can be verified with examples. These calculations should be connected back to the context of the problem.

NC Math 3

Focus:
The focus of *Statistics and Probability* in NC Math 3 is on the use of sample data to represent a population. Important understandings include the necessity of randomization in sampling to ensure an unbiased representation of a population. Students will engage in simulation to create a random sample of a population, calculating sample statistics representative of population statistics. Students will examine and evaluate the design and results of population studies that utilize samples.

Rationale:
Students need to be informed about the appropriate design and implementation of studies that utilize sample statistics to represent a population. This course will introduce students to the process of

inference: that is, drawing conclusions about a population based on information from a sample. Students will use simulation to see that the variability in samples can be used to predict a margin of error. This work will be based on simulation only, not on any computational formulas. By the end of the course students should understand how statistics is used in the real world to analyze data and make decisions.